THE EFFECTIVE MATERIAL PARAMETERS OF REAL MEDIA

Focus on seismic fracture characterization in the presence of anisotropic permeability

A presentation by Morten Jakobsen for members and friends of the new CIPR, 31.01.03.

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OUTLINE OF THE TALK

1. **Introduction.**
2. **Stochastic wave propagation.**
3. **Fluid dynamical considerations.**
4. **Numerical examples.**
5. **Concluding remarks.**

1. Introduction

- **Enhanced reservoir characterization by seismic modelling requires a mathematical model that accounts for the effects of microstructure and fluid flow on the overall wave characteristics.**
- **A unified model of rocks as viscoelastic composites have recently been developed.**
- **We have used a combination of stochastic integral equation methods and fluid dynamic considerations involving several length scales.**

- It is important to focus on (aligned) cracks/fractures because they tend control the flow of fluids (in different directions).
- Cracks/fractures can be characterized with seismics because they
  - decrease P- and S-wave velocity.
  - increase velocity dispersion and wave attenuation.
  - increase pressure-dependence of velocity/attenuation.
  - increase velocity and attenuation anisotropy.
  - increase potential for stress-induced anisotropy.

200 *T. Pointer, E. Liu and J. A. Hudson*

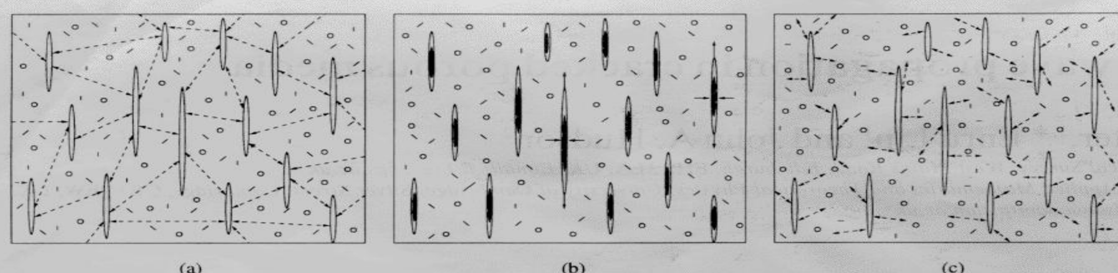


Figure 1. A schematic 2-D diagram that shows the three different models for fluid flow: (a) between connected cracks, (b) within cracks partially saturated with two different fluids, and (c) from cracks into a background porous matrix. The cracks are aligned in this diagram but have random orientations.

2. Stochastic wave propagation

- **The (time-reduced) constitutive relation for a visco-elastic continuum:**

$$\Phi(\mathbf{x}) = \mathcal{R}(\mathbf{x})\Psi(\mathbf{x}) \tag{1}$$

- **The stress-momentum vector:**

$$\Phi(\mathbf{x}) = [\boldsymbol{\sigma}(\mathbf{x}), \mathbf{p}(\mathbf{x})]^T \tag{2}$$

- **The strain-velocity vector:**

$$\Psi(\mathbf{x}) = [\boldsymbol{\epsilon}(\mathbf{x}), -i\omega\mathbf{u}(\mathbf{x})]^T \tag{3}$$

- **The stiffness-density matrix:**

$$\mathcal{R}(x) = \begin{pmatrix} \mathbf{C}(\mathbf{x}) & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\rho}(\mathbf{x}) \end{pmatrix} \tag{4}$$

- **The effective constitutive relation for the (statistically homogeneous) medium as a whole:**

$$\langle \Phi(\mathbf{x}) \rangle = \mathcal{R}^* \langle \Psi(\mathbf{x}) \rangle \tag{5}$$

- $\langle \Phi(\mathbf{x}) \rangle$ **is the ensemble-averaged stress-momentum vector.**
- $\langle \Psi(\mathbf{x}) \rangle$ **is the ensemble-averaged strain-velocity vector.**
- **The problem is to determine the effective stiffness-density matrix $\mathcal{R}^*$ by using the statistical information we have about $\mathcal{R}(\mathbf{x})$.**

- **A 'dynamic equilibrium' condition representing the EOM:**

$$\nabla_4 \cdot \Phi(\mathbf{x}) = 0 \tag{6}$$

- **Generalized gradient operator:**

$$\nabla_4 \equiv [\nabla, -i\omega] \tag{7}$$

- **Decomposition of the stiffness-density matrix:**

$$\mathcal{R}(\mathbf{x}) = \mathcal{R}^{(0)} + \delta\mathcal{R}(\mathbf{x}) \tag{8}$$

- **An arbitrary homogeneous reference medium:**

$$\mathcal{R}^{(0)} = \begin{pmatrix} \mathbf{C}^{(0)} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\rho}^{(0)} \end{pmatrix} \tag{9}$$

- **The corresponding fluctuation:**

$$\delta\mathcal{R}(\mathbf{x}) = \begin{pmatrix} \delta\mathbf{C}(\mathbf{x}) & 0 \\ 0 & \delta\boldsymbol{\rho}(\mathbf{x}) \end{pmatrix} \tag{10}$$

- **A Lippmann-Schwinger-Dyson (LSD) type of integral equation for the local motion:**

$$\Psi(\mathbf{x}) = \Psi^{(0)}(\mathbf{x}) + \int d\mathbf{x}' \mathcal{G}^{(0)}(\mathbf{x}-\mathbf{x}')\delta\mathcal{R}(\mathbf{x}')\Psi(\mathbf{x}') \tag{11}$$

- $\Psi^{(0)}(\mathbf{x})$ **is the strain-velocity vector associated with** $\mathcal{R}^{(0)}$**.**
- **Generalized Green's function:**

$$\mathcal{G}^{(0)}(\mathbf{x}) = \begin{pmatrix} \mathbf{S}_x^{(0)}(\mathbf{x}) & \mathbf{M}_x^{(0)}(\mathbf{x}) \\ \mathbf{S}_t^{(0)}(\mathbf{x}) & \mathbf{M}_t^{(0)}(\mathbf{x}) \end{pmatrix} \tag{12}$$

- **Here** $\mathbf{S}_x^{(0)}(\mathbf{x})$, $\mathbf{M}_x^{(0)}(\mathbf{x})$, $\mathbf{S}_t^{(0)}(\mathbf{x})$ **and** $\mathbf{M}_t^{(0)}(\mathbf{x})$ **are modified Green's functions for the reference material.**

- **Exact formal solution to the effective medium problem:**

$$\mathcal{R}^* = \mathcal{R}^{(0)} + \langle \mathcal{T} \rangle \left[\mathcal{I} + \bar{\mathcal{G}} \langle \mathcal{T} \rangle \right]^{-1} \tag{13}$$

- **The (yet to be determined) $\mathcal{T}$ matrix for the material also satisfies a LSD-type of integral equation:**

$$\mathcal{T}(\mathbf{x}) = \delta\mathcal{R}(\mathbf{x}) + \delta\mathcal{R}(\mathbf{x}) \int d\mathbf{x}' \tilde{\mathcal{G}}(\mathbf{x} - \mathbf{x}') \mathcal{T}(\mathbf{x}') \tag{14}$$

- **A spatial invariant Greens matrix:**

$$\bar{\mathcal{G}} = \int d\mathbf{x}' \tilde{\mathcal{G}}(\mathbf{x} - \mathbf{x}') \tag{15}$$

- **A transformed generalized Green's function:**

$$\tilde{\mathcal{G}}(\mathbf{x} - \mathbf{x}') = \mathcal{G}^{(0)}(\mathbf{x} - \mathbf{x}') e^{i\mathbf{k}\cdot(\mathbf{x}' - \mathbf{x})} \tag{16}$$

3. Fluid dynamical considerations

- **If $\tilde{v}^{(n)}$ and $\rho_f^{(n)}$ is the porosity and density of the nth cavity set, respectively, then the total fluid mass m_f is given by**

$$m_f = \sum_{r=1}^{N_c} \tilde{v}^{(r)} \rho_f^{(r)} \tag{17}$$

- **We now require that the fluid mass in an arbitrary volume is conserved and that the average flow of fluid is regulated by Darcy's law, so that**

$$\frac{\partial m_f}{\partial t} = \nabla \cdot \left(\frac{\rho_f}{\eta_f} \mathbf{\Gamma} \cdot \nabla p_f \right) \tag{18}$$

where p_f is the average (local) fluid pressure, ρ_f is the fluid mass density, η_f is the viscosity of the fluid, and $\mathbf{\Gamma}$ is a second-rank tensor of permeability parameters. The tensor $\mathbf{\Gamma}$ represents the overall permeability of the material (including all cavities) and is assumed to be spatially invariant.

- **The fluid pressure and density of the nth cavity set are related by**

$$\frac{\rho_0}{\rho_f^{(n)}} = 1 - \frac{p_f^{(n)}}{\kappa_f} \tag{19}$$

where ρ_0 is the density of the unstressed fluid, and κ_f is the fluid bulk modulus.

- **If a quasi-static stress field is imposed on the macroscopic specimen then the pressure $p_f^{(n)}$ in the fluid changes, due both to a change in porosity and to fluid flow.**

- **We have derived a higher-order expression for the change in porosity:**

$$\frac{\tilde{v}^{(n)} - v^{(n)}}{v^{(n)}} = \left(\hat{K}_d^{(n)}\right)_{uupq} \left(\sigma_{pq}^{(0)} + \delta_{pq} p_f^{(n)}\right) - S_{uupq}^{(0)} \delta_{pq} p_f^{(n)}, \tag{20}$$

where $v^{(n)}$ is the unstressed porosity of the nth cavity set.

- **We assume that the mass flow out of the nth set of cavities is controlled by an expression similar to that of Hudson et al. (1996):**

$$\frac{\partial\left(\rho_f^{(n)} \tilde{v}^{(n)}\right)}{\partial t} = -\frac{v^{(n)} \rho_0}{\kappa_f \tau} \left(p_f^{(n)} - p_f\right), \tag{21}$$

where τ is a (squirt flow) relaxation time constant, which is proportional to the fluid viscosity η_f and inversely proportional to a permeability constant.

- **A novel approximation for the effective compliance tensor of complex porous media:**

$$\mathbf{S}^* = \mathbf{S}^{(0)} + \sum_r v^{(r)}\mathbf{K}_d^{(r)} - \sum_r \frac{v^{(r)}\mathbf{K}_d^{(r)}}{1 + i\omega\gamma^{(r)}\tau} : (\mathbf{I}_2 \otimes \mathbf{I}_2) : \left(\Theta(\omega) \sum_s \frac{v^{(s)}\mathbf{K}_d^{(s)}}{1 + i\omega\gamma^{(s)}\tau} + i\omega\tau\kappa_f \mathbf{K}^{(r)}\right) \quad (22)$$

- **A frequency-dependent quantity reflecting the interconnected pores and cracks:**

$$\Theta(\omega) = \kappa_f \left\{ \left(1 - \kappa_f S_{uuvv}^{(0)}\right) \sum_{r=1}^{N_c} \frac{v^{(r)}}{1 + i\omega\gamma^{(r)}\tau} + \kappa_f \sum_{r=1}^{N_c} \frac{v^{(r)} \left(K_d^{(r)}\right)_{uuvv}}{1 + i\omega\gamma^{(r)}\tau} - \frac{ik_u k_v \Gamma_{uv}\kappa_f}{\eta_f \omega} \right\}^{-1} \quad (23)$$

- **A frequency-independent quantity reflecting the response of a single cavity:**

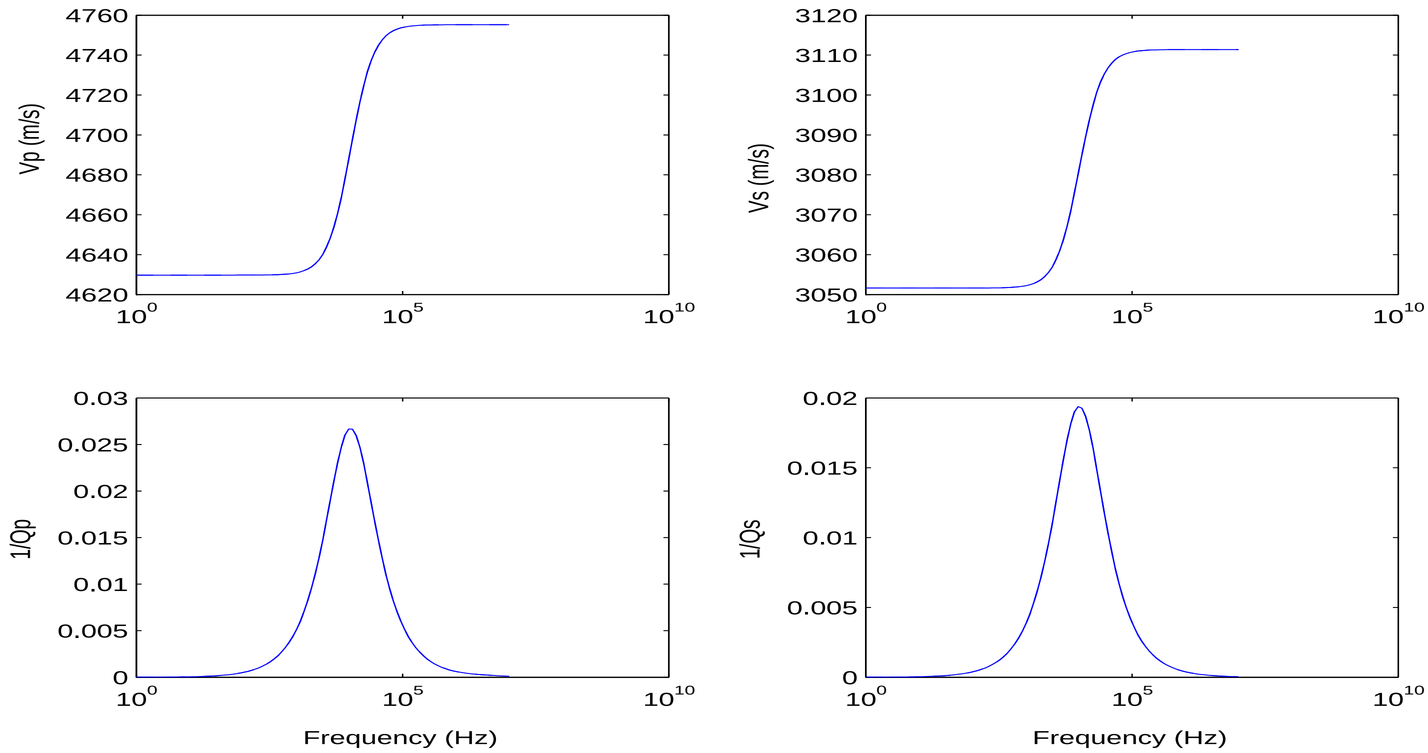
$$\gamma^{(n)} = 1 + \kappa_f \left(K_d^{(n)} - S^{(0)}\right)_{uuvv} \quad (24)$$

- **We have shown analytically that the above formulae is consistent with the classic result of Gassmann (1951).**

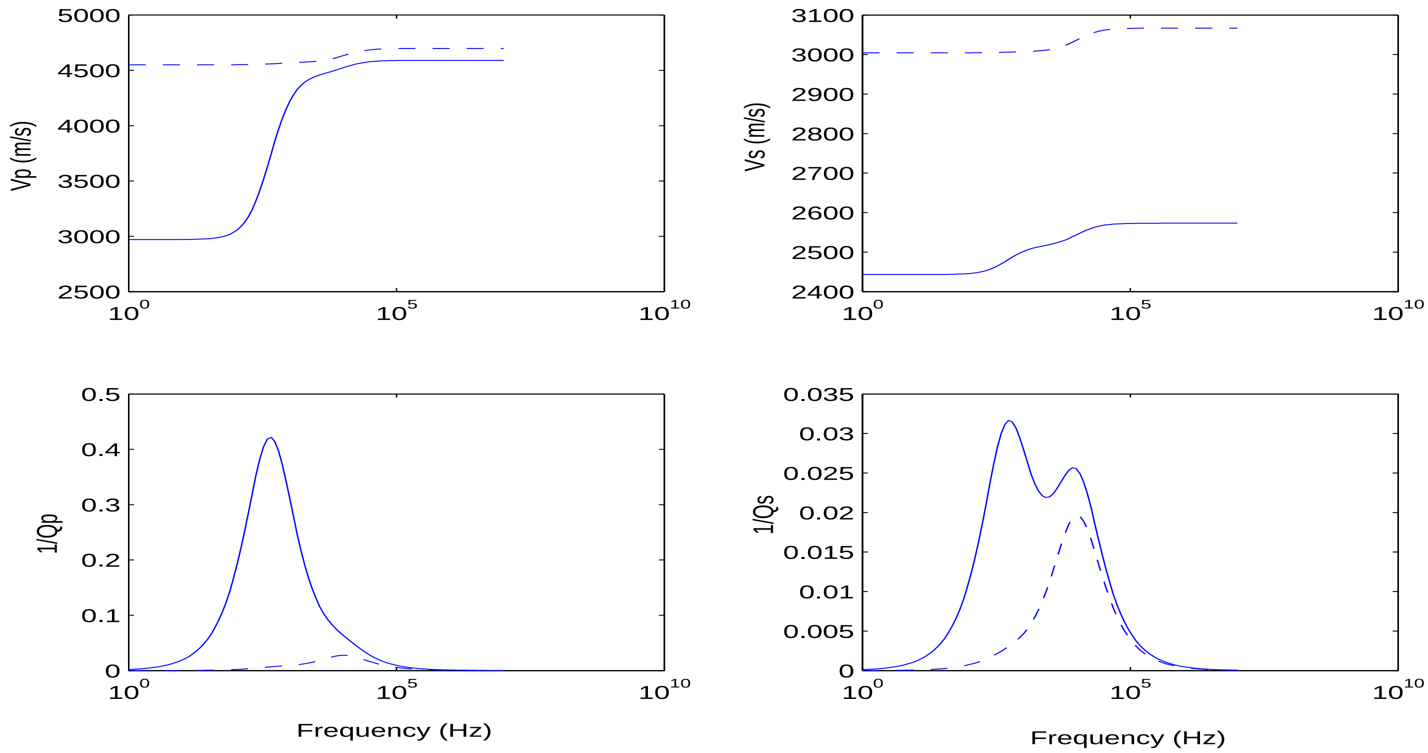
4. Numerical examples

- **Crack-induced anisotropy.**
- **Shear-wave splitting.**
- **The effect of crack-size.**

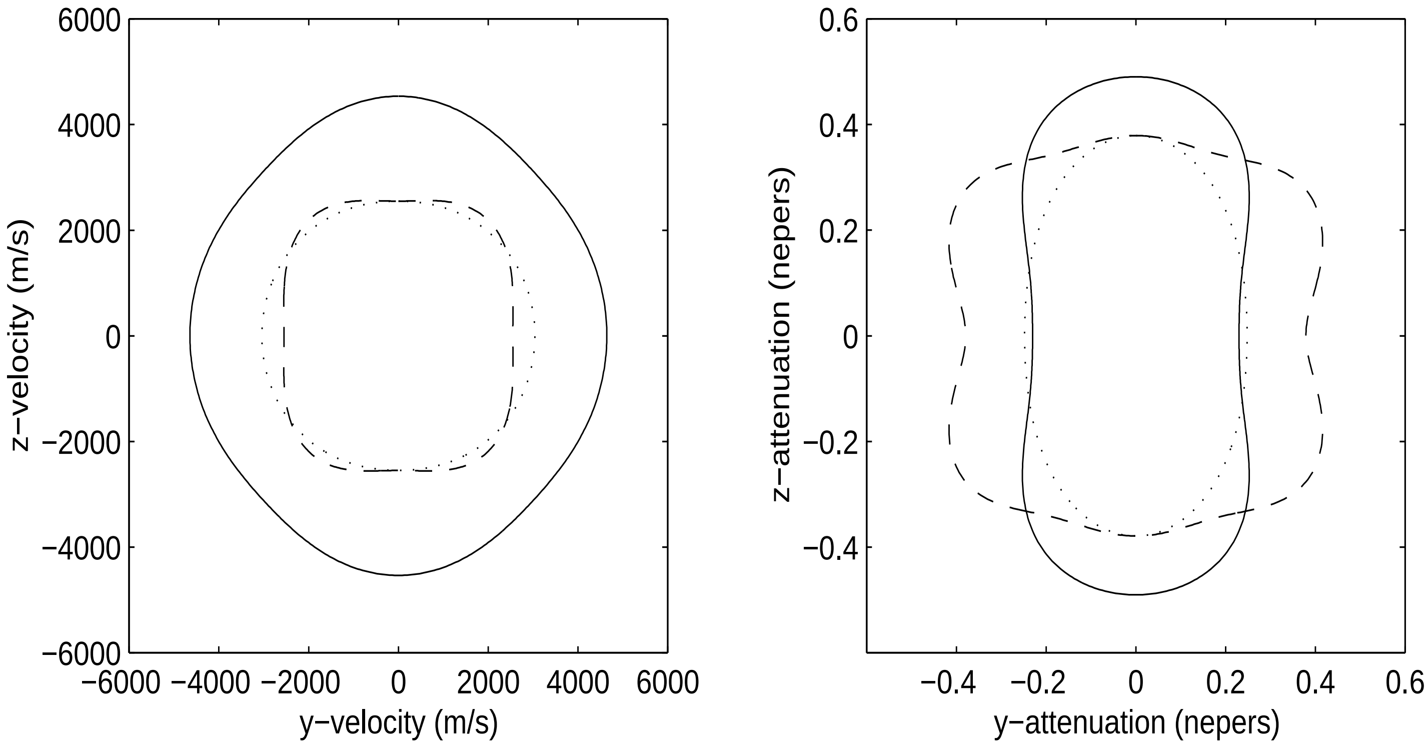
Isotropic reference medium.



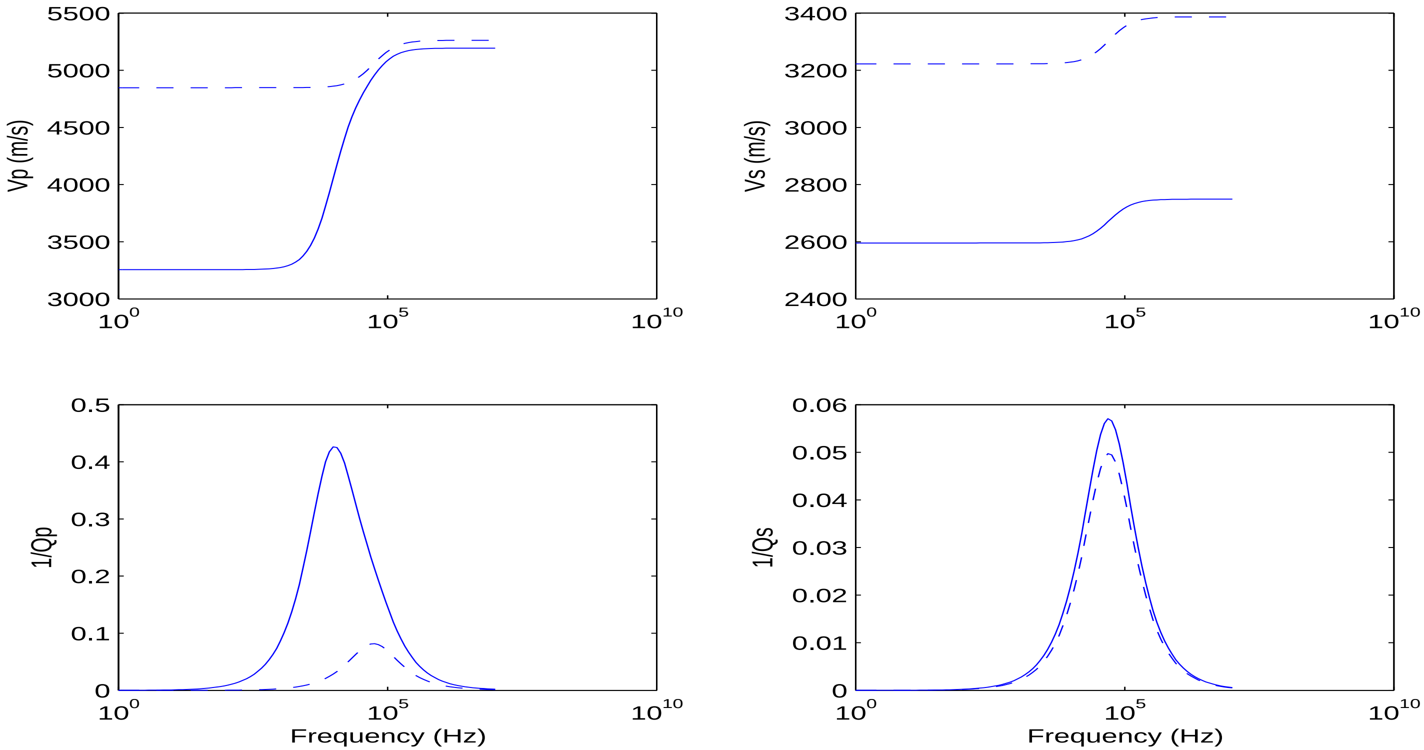
Anisotropic medium containing nearly aligned cracks.



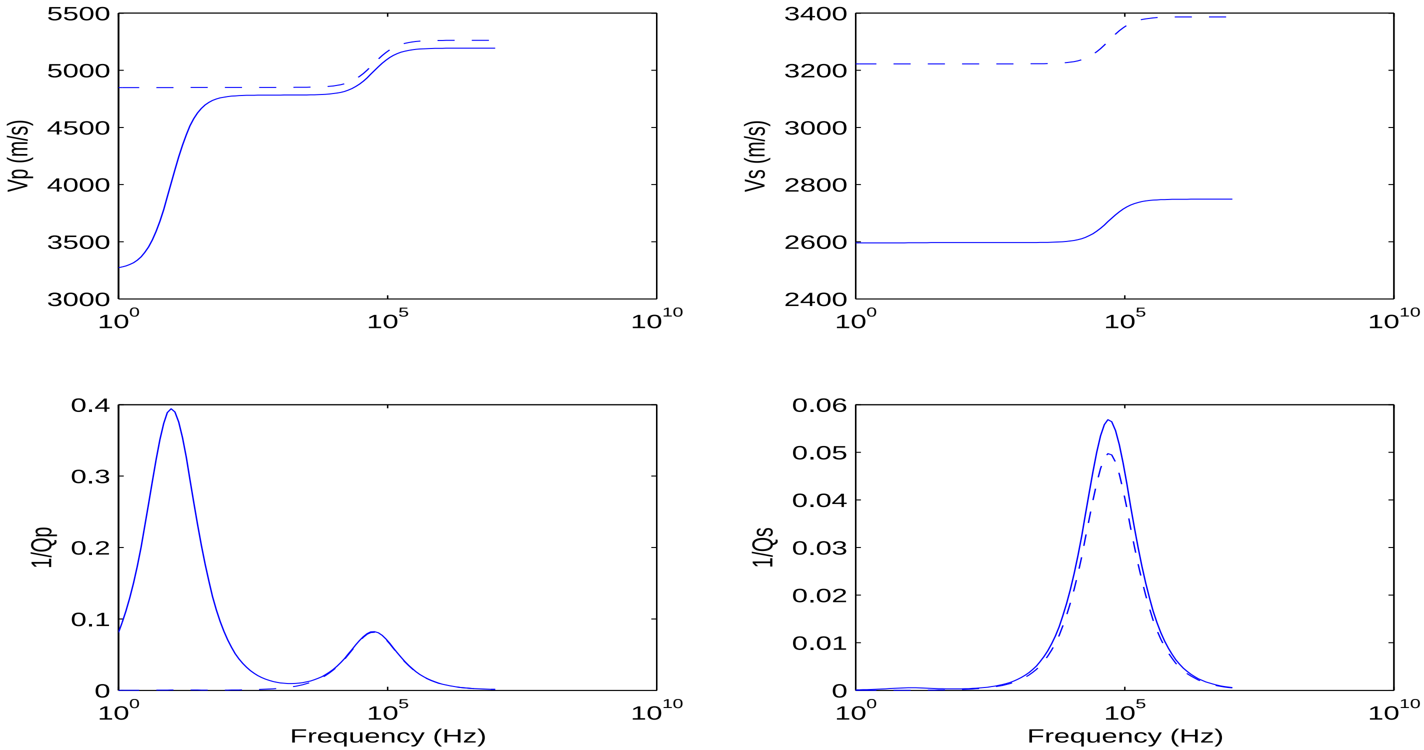
The phase velocity and attenuation surfaces of a cracked porous medium of hexagonal symmetry:

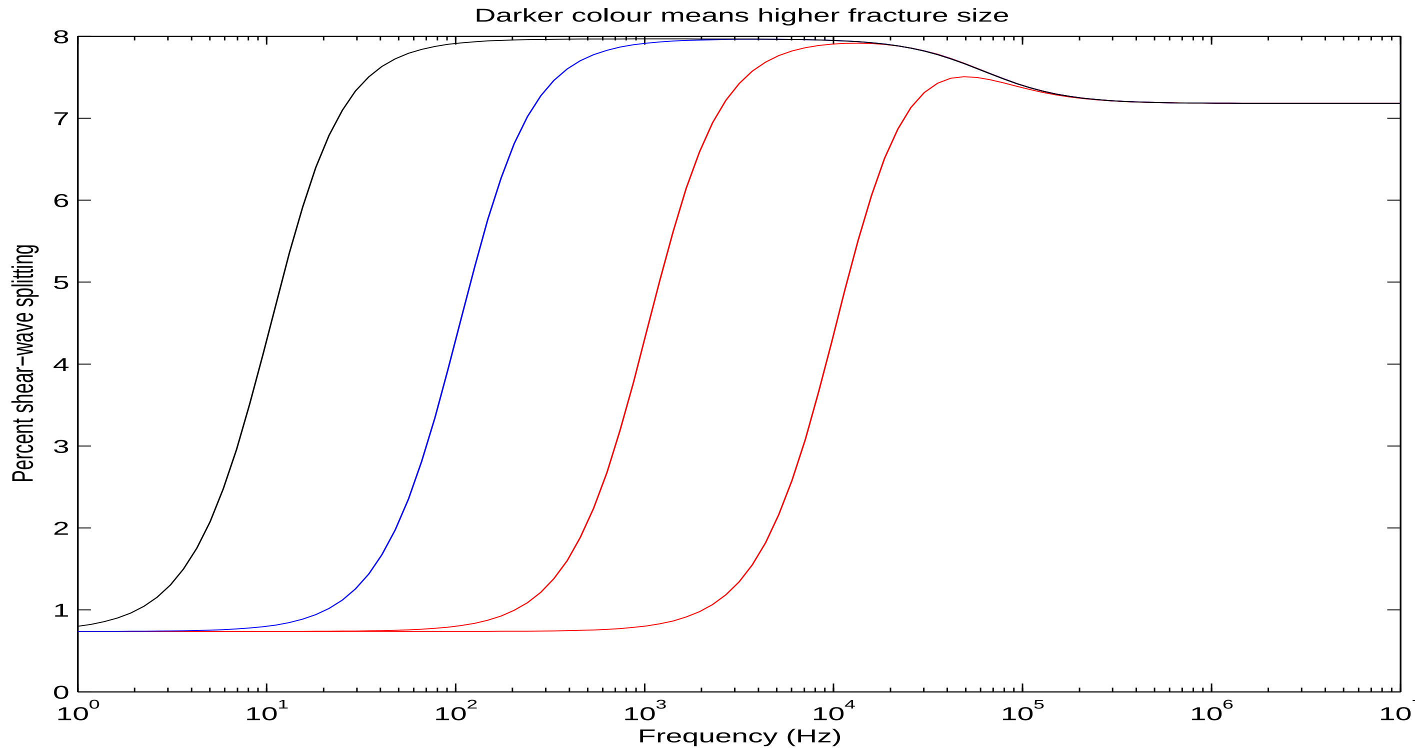


Fracture size = 0.1 cm.



Fracture size = 100.0 cm.




Darker colour means higher fracture size
Percent shear-wave splitting
Frequency (Hz)
0
1
2
3
4
5
6
7
8
10^0
10^1
10^2
10^3
10^4
10^5
10^6
10^7

5. CONCLUDING REMARKS

- **We have seen that rock physics is**
  - **an evolving science that calls for an interdisiplinary perspective.**
  - **an art in the sense that creativity is needed when modelling extremely complex systems.**
  - **an important tool for seismic reservoir characterization.**