Inversion of reflection seismic data constrained by well log data

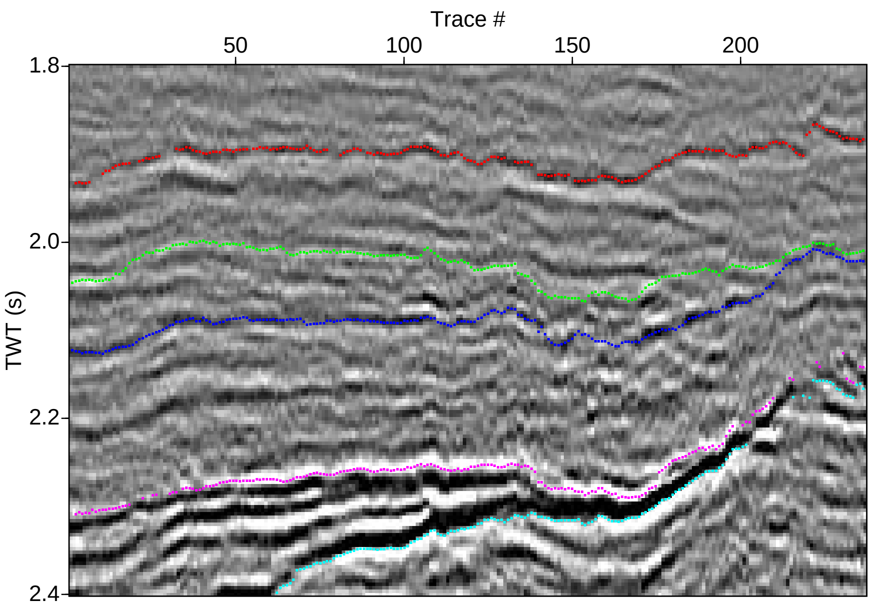
Background

- **Inversion of reflection seismic data for P- and S-velocity and density is an inherently under-determined problem.**
- **In areas where well logs are available this information can be used to constrain the inversion.**
- **An algorithm for seismic inversion for the properties of a plane layered model have been developed using raytracing for forward modelling.**

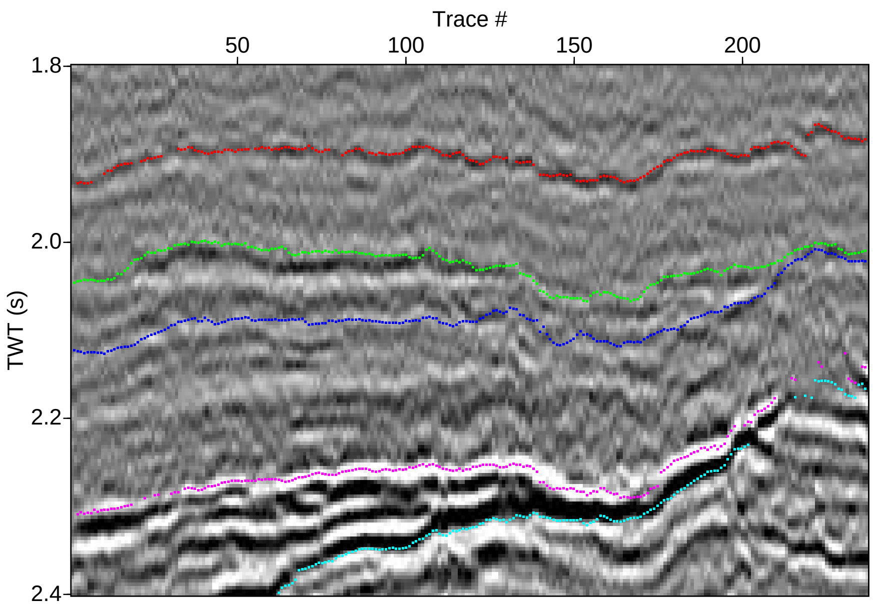
Outline of talk

- **Data used (reduced offset stacks, sonic and density logs)**
- **Method (inversion methods including *a priori information*)**
- **Results (earth models and synthetic seismic sections)**
- **Conclusions and strategy for further development**

Reduced offset stacks (3D dataset)



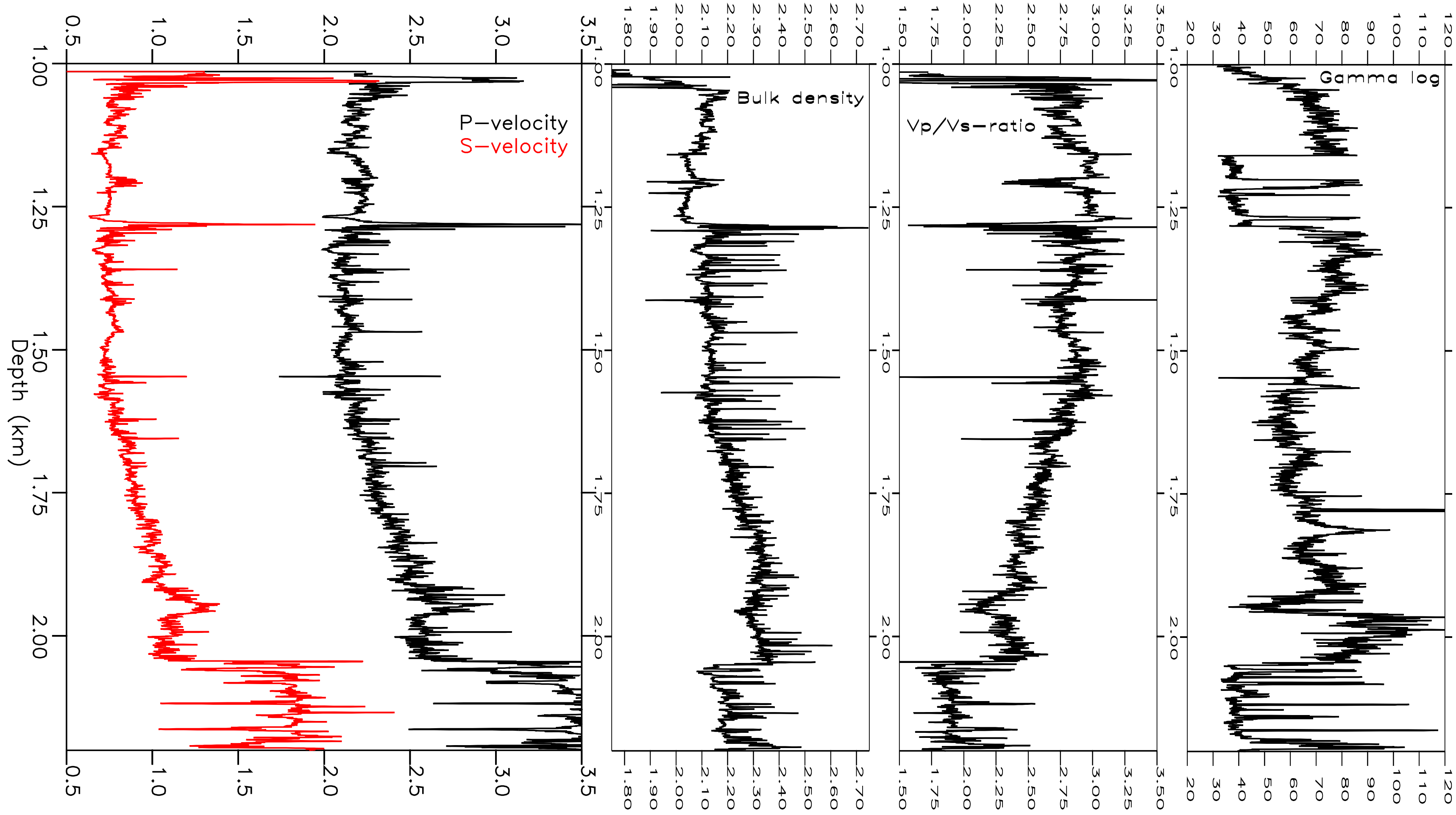
Near stack 3D line



Far stack 3D line

Near stack: 0–600 m offset, Far stack: offsets > 1200 m (maximum offset at 2.0 s TWT is 2200 m).

Well log data

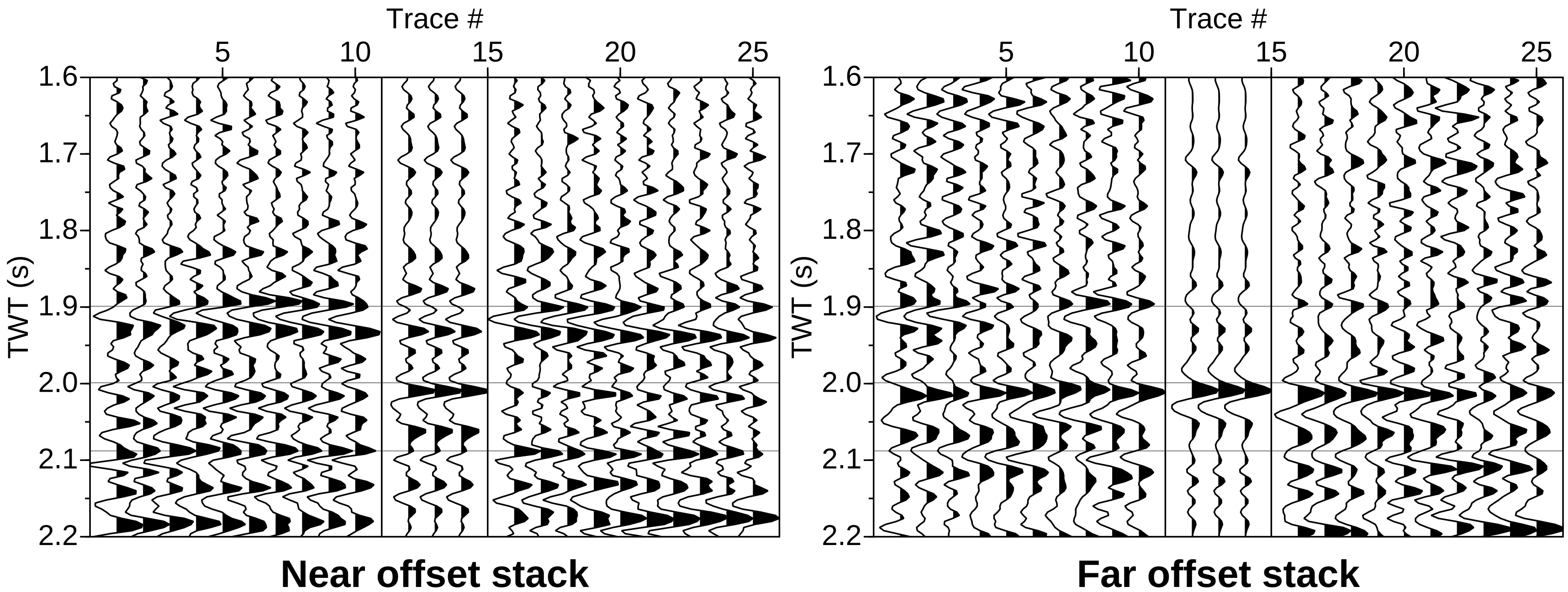


Seismic modelling of well log data

- **Checkshot corrections**
- **Correcting for mud invation**
- **Blocking logs into layers (upscaling by Backus averaging)**
- **Full elastic wavefield modelling by reflectivity method**
- **Normal moveout correction and stacking with reduced offset intervals**
- **Comparing with recorded seismic data**

Comparing modelled data with seismic section

The three middle traces are stacked synthetic data inserted into the real seismic sections.



Obviously, there are problems to match the real data even with well log data available.

Inversion with *a priori* information

A maximum likelihood solution is found by minimizing the objective funcion

$$f(\mathbf{m}) = \frac{1}{2}[G(\mathbf{m}) - \mathbf{d}]^T C_d^{-1}[G(\mathbf{m}) - \mathbf{d}] + \frac{1}{2}[\mathbf{m} - \mathbf{m_0}]^T C_m^{-1}[\mathbf{m} - \mathbf{m_0}]$$

m : model – Consists of P and S velocities and densities, layer boundaries are fixed

$\mathbf{m_0}$: *a priori* model determined from well logs

d : data – seismic traces recorded at several offsets, may also be reduced offset stacks

$G(\mathbf{m})$: synthetic traces computed from model m

C_m^{-1} : *a priori* covariance matrix for model

C_d^{-1} : *a priori* covariance matrix for data

C_m is assumed to be a diagonal matrix depending only on the *a priori* variances of the P and S velocities and densities.
C_d is also assumed to be diagonal and to depend only of the data variance, $C_d = \sigma_d^2 I$.

Minimization procedure - gradient method

Compute

$$\mathbf{g_k} = \nabla f(\mathbf{m_k})$$

and iterate

$$\mathbf{m_{k+1}} = \mathbf{m_k} - \alpha_k \mathbf{g_k}.$$

The gradient is given by

$$\nabla f(\mathbf{m}) = [\nabla G(\mathbf{m})]^T C_d^{-1} [G(\mathbf{m}) - \mathbf{d}] + C_m^{-1} [\mathbf{m} - \mathbf{m_0}]$$

and the step length α_k is found by searching in the gradient direction.

Advantages: Fast computations (no matrix inversion), relatively easy to implement, stable.

Disadvantages: Convergence may be slow, may converge into a local minimum.

Inversion method applied to seismic waveform data

Denoting the recorded seismograms at receiver i $(i = 1, .., N)$ as $d_i(t)$ and the corresponding synthetic seismograms as $s_i(\mathbf{m}, t)$, the objective function may be written

$$f(\mathbf{m}) = \frac{1}{2}\sigma_d^{-2}\sum_{i=1}^{N}\int_{t_1}^{t_2}[s_i(\mathbf{m}, t) - d_i(t)]^2 dt + \frac{1}{2}\sum_{i=1}^{M}(m_i - m_{0i})^2/\sigma_{mi}^2$$

where t_1 and t_2 are limits of the chosen time window and M is the number of model parameters.

The components of the gradient are then computed from

$$\frac{\partial f}{\partial m_j} = \sigma_d^{-2}\sum_{i=1}^{N}\int_{t_1}^{t_2}\frac{\partial s_i(\mathbf{m}, t)}{\partial m_j}[s_i(\mathbf{m}, t) - d_i(t)]dt + \sum_{i=1}^{M}(m_i - m_{0i})/\sigma_{mi}^2$$

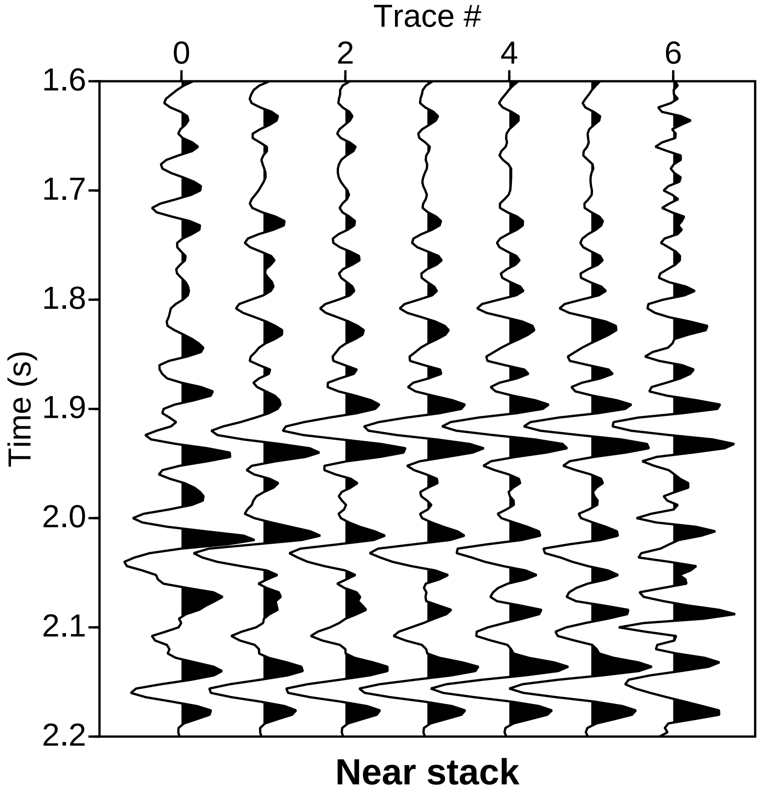
where $[s_i(\mathbf{m}, t) - d_i(t)]$ is the data residuals and

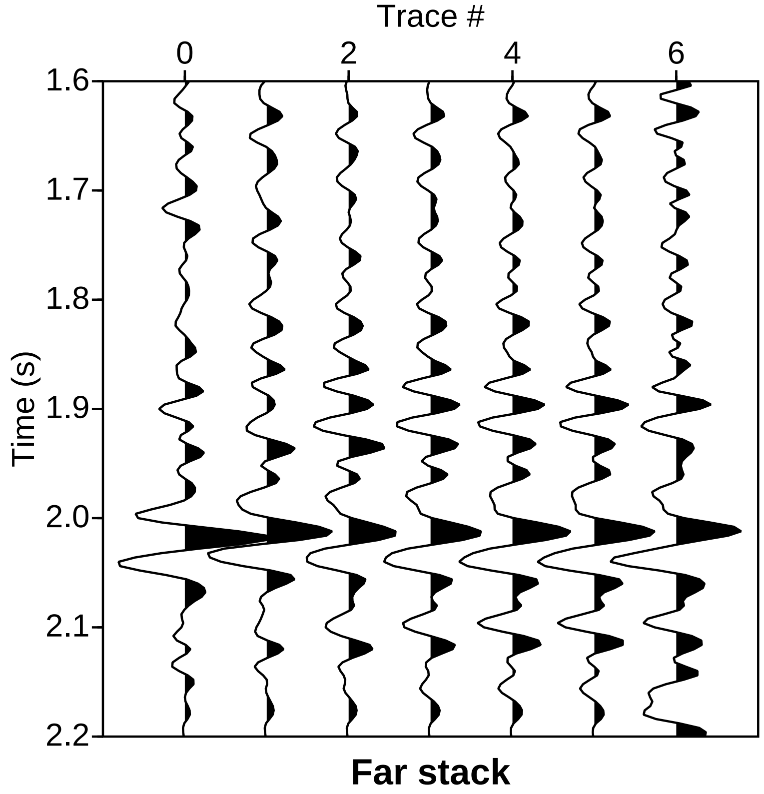
$$\frac{\partial s_i(\mathbf{m}, t)}{\partial m_j} \approx \frac{s_i(\mathbf{m} + \delta m_j, t) - s_i(\mathbf{m}, t)}{\delta m_j}$$

is called the differential seismograms.

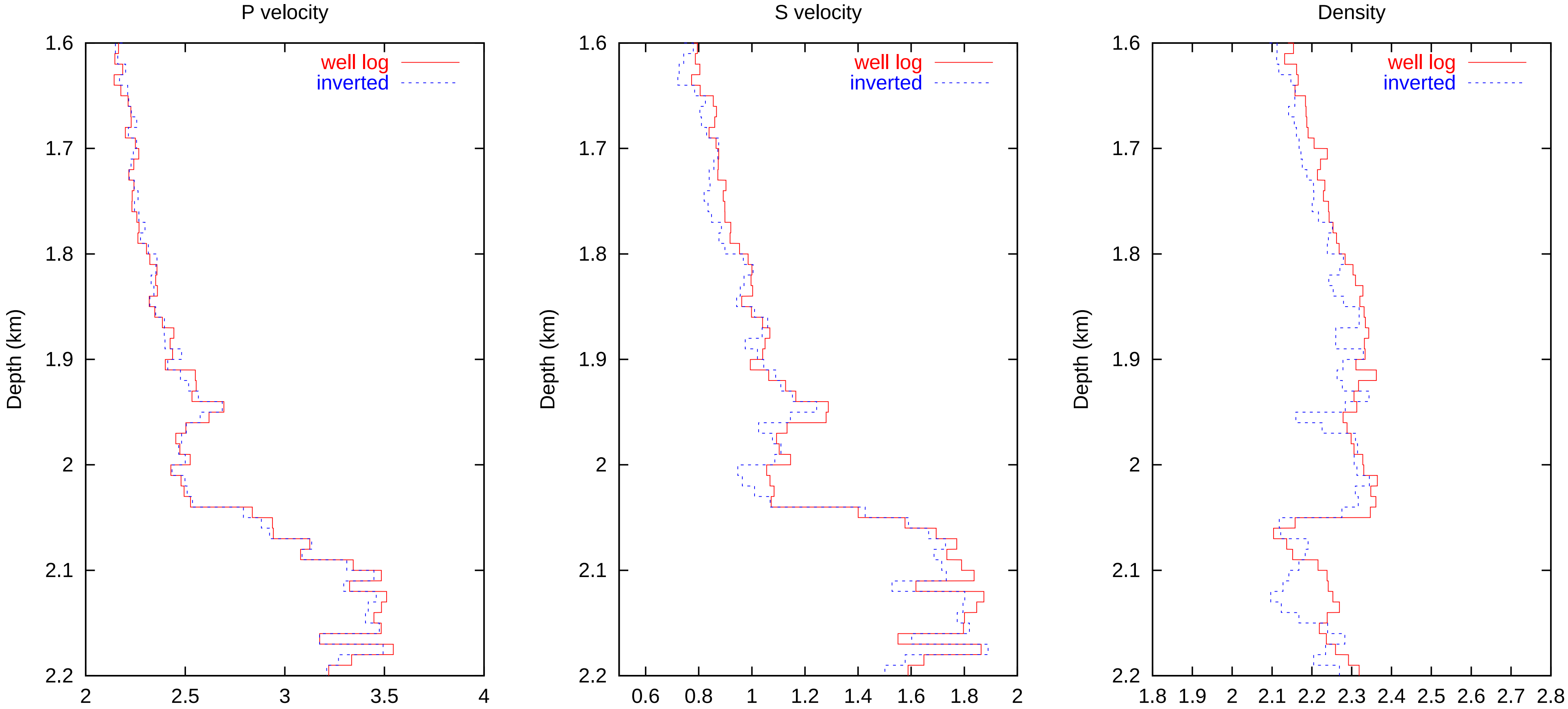
Inversion results

The first trace in each section is the synthetic trace computed from the *a priori* model, and the last trace is the observed data. In between are shown synthetic traces from each iteration of the inversion procedure.



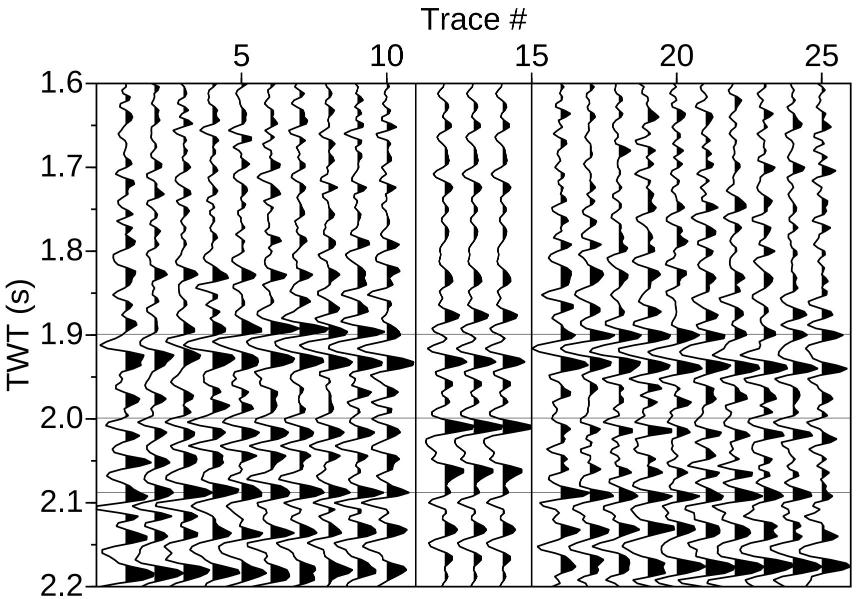


Inversion results

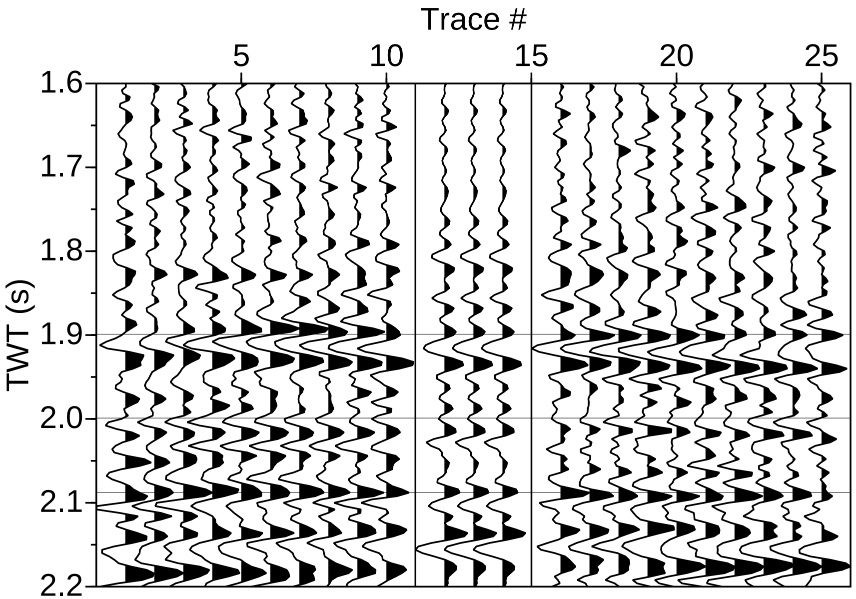


Comparing modelled data with seismic section
Near offset stacks

The three middle traces are stacked synthetic data inserted into the real seismic sections.



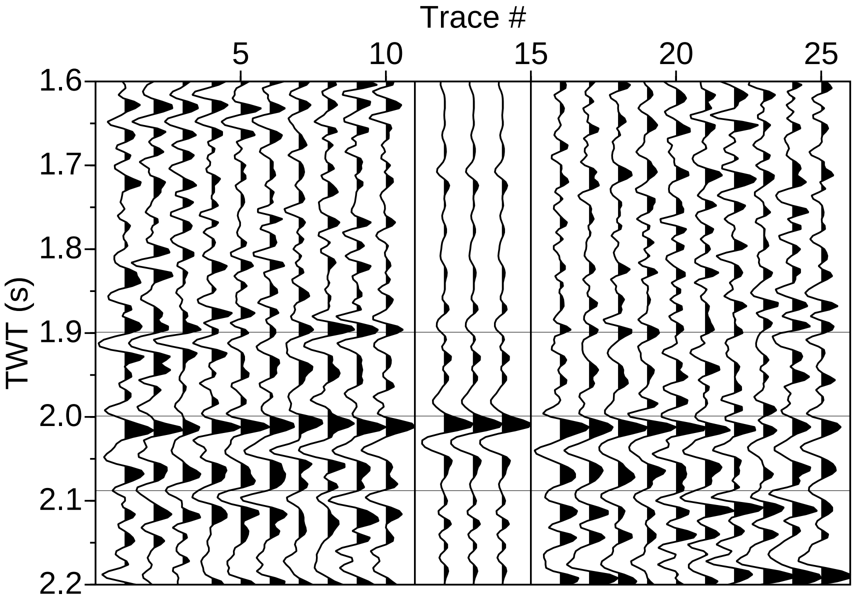
Near offset stack before inversion



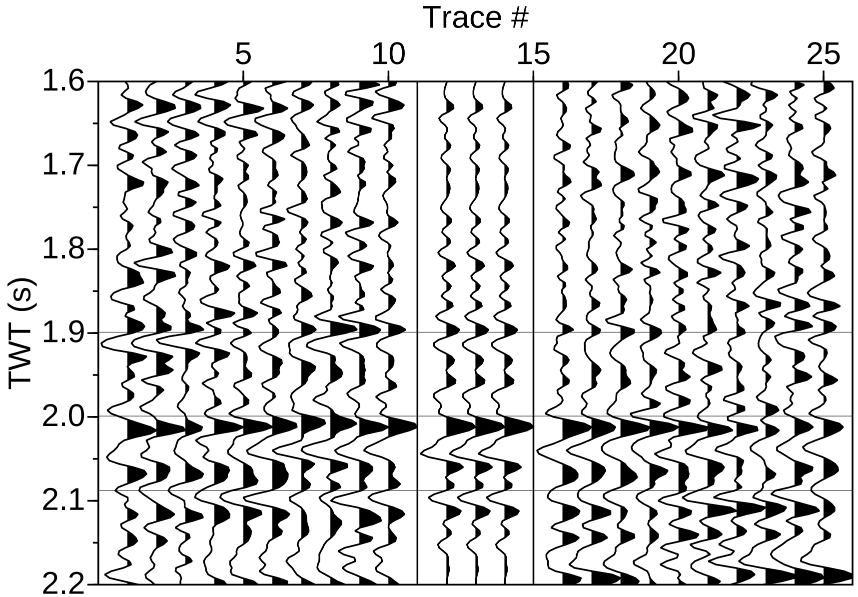
Near offset stack after inversion

Comparing modelled data with seismic section
Far offset stacks

The three middle traces are stacked synthetic data inserted into the real seismic sections.



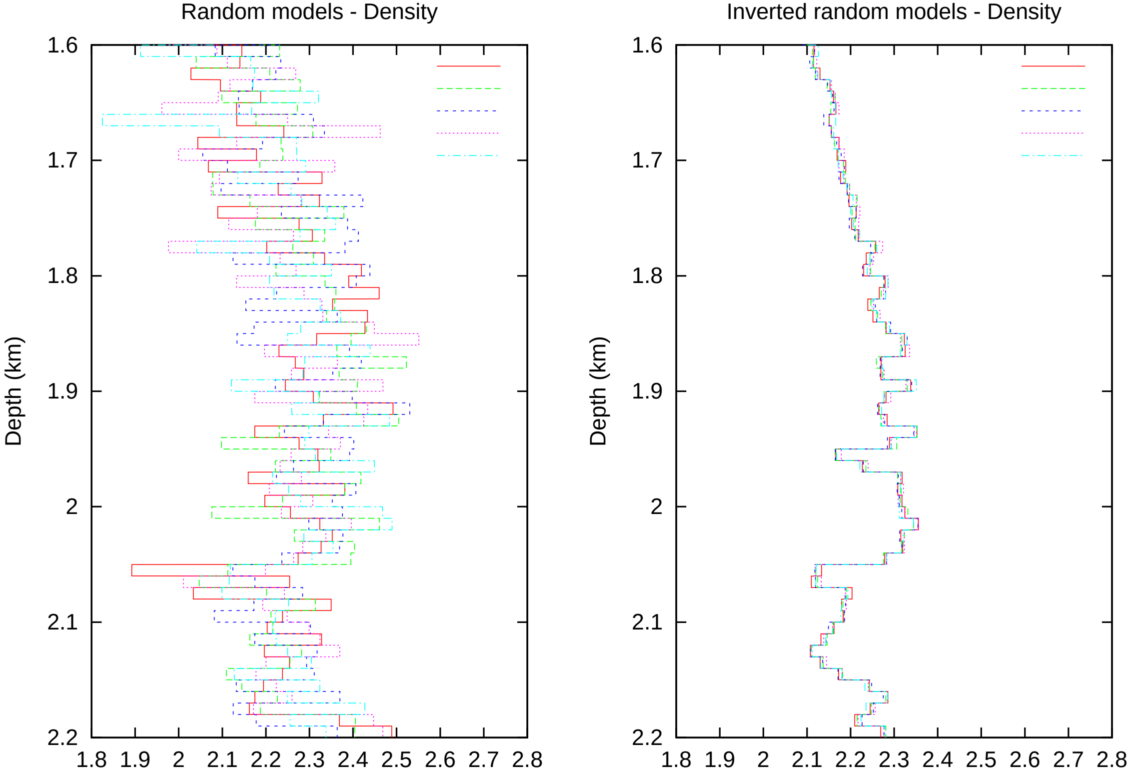
Far offset stack before inversion



Far offset stack after inversion

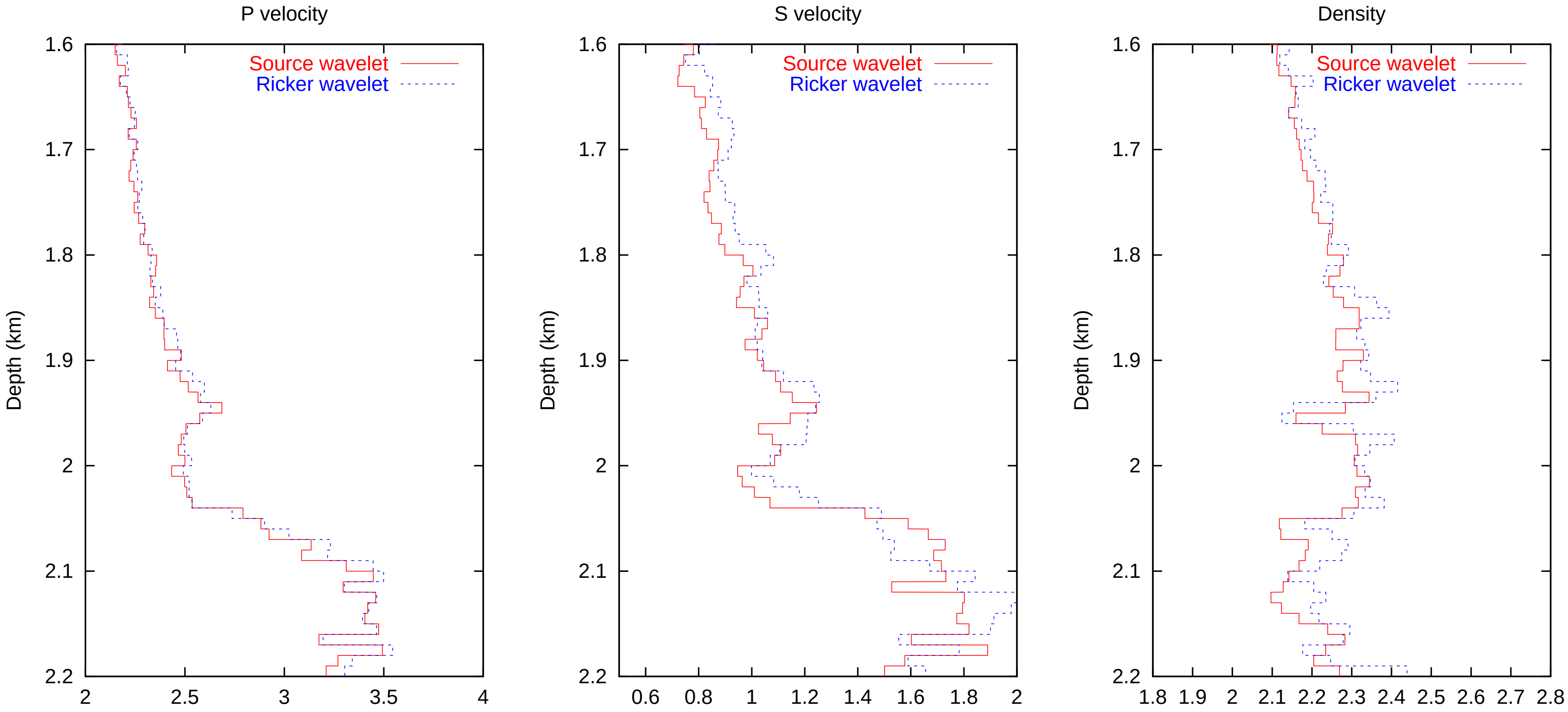
Inversion with different starting models

A known problem with the gradient method is that it may converge into a local minimum. To check this the inversion was performed with several randomly perturbed starting models.

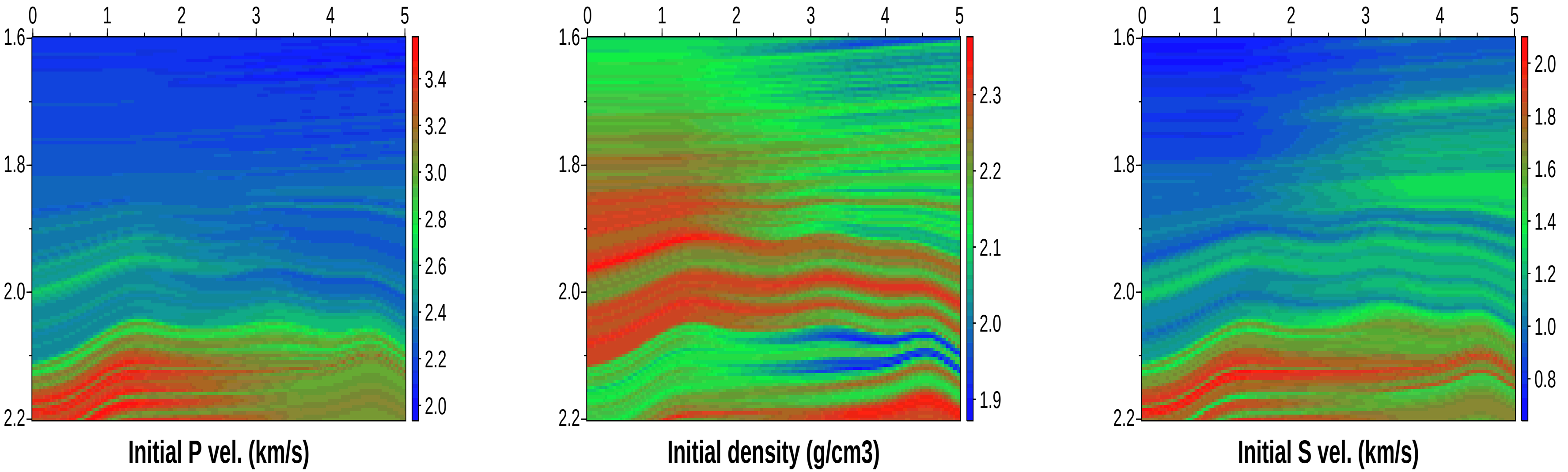


Inversion with different source wavelets

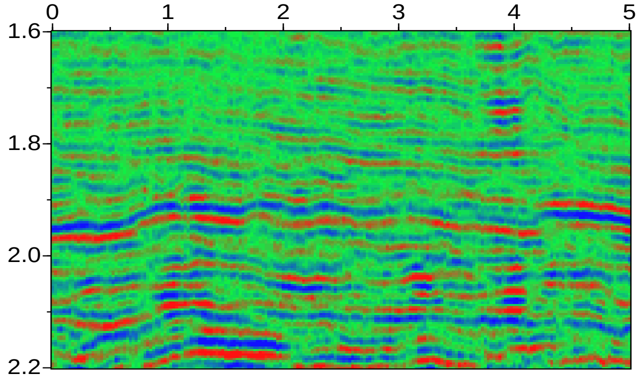
In the inversion procedure it is assumed that the source wavelet is known.
What is the effect of using the wrong wavelet ?
In our case the provided source wavelet resembles a Ricker wavelet with a 90° phase shift.



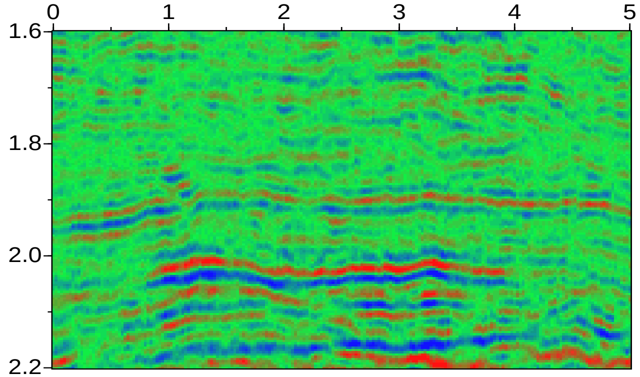
Seismic model interpolated from well logs



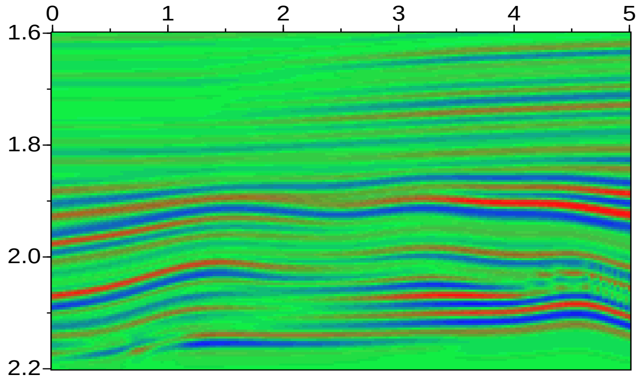
Synthetics from interpolated model



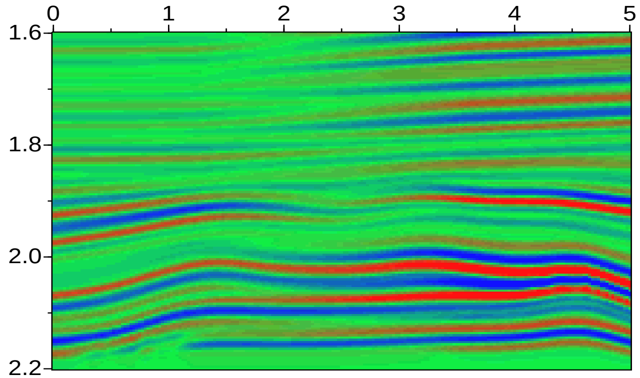
Recorded near stack



Recorded far stack

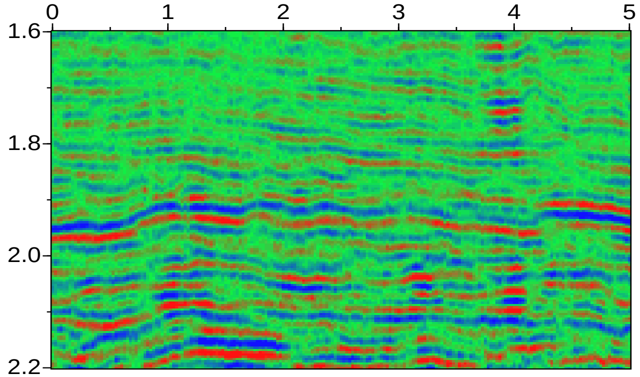


Near synt. initial mod.

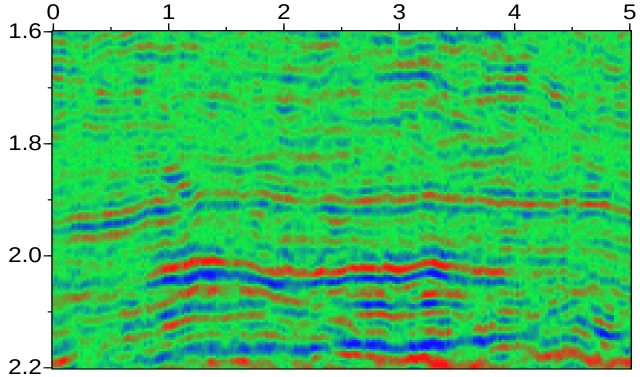


Far synt. initial mod.

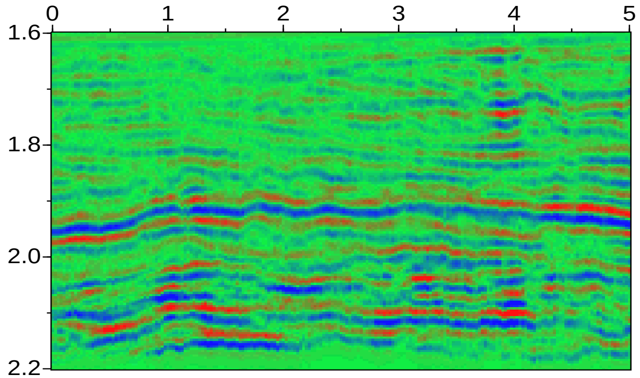
Synthetics from inverted model



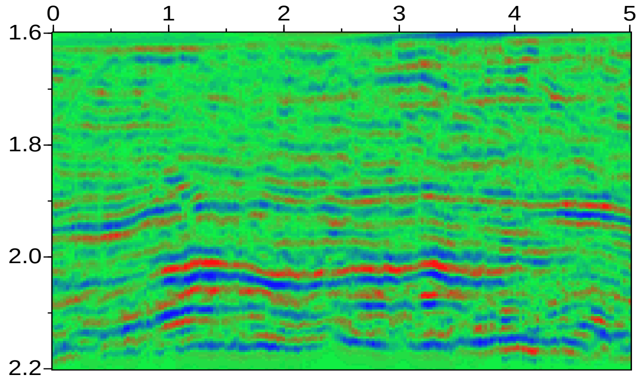
Recorded near stack



Recorded far stack

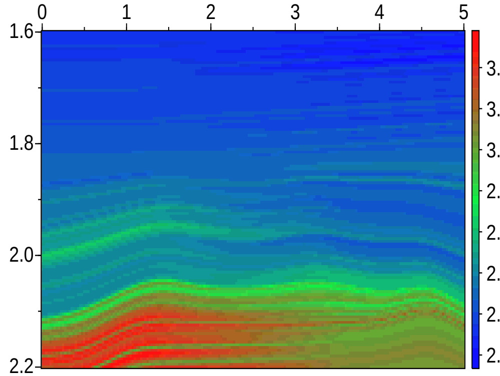


Near synt. final mod.

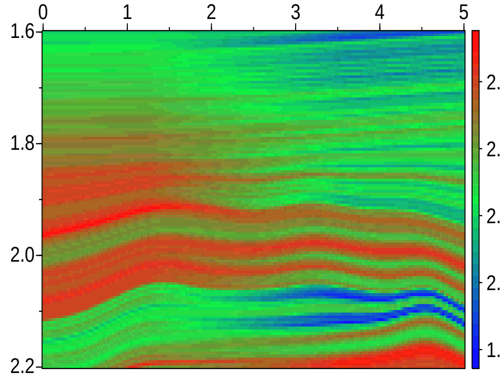


Far synt. final mod.

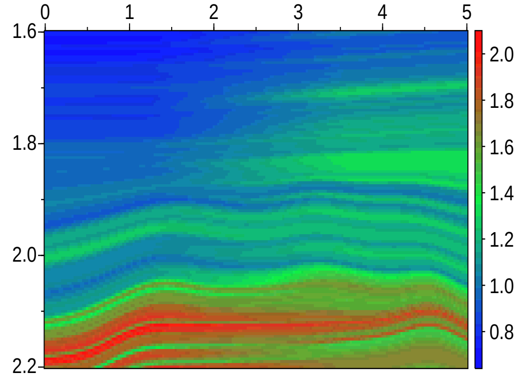
Inverted seismic model



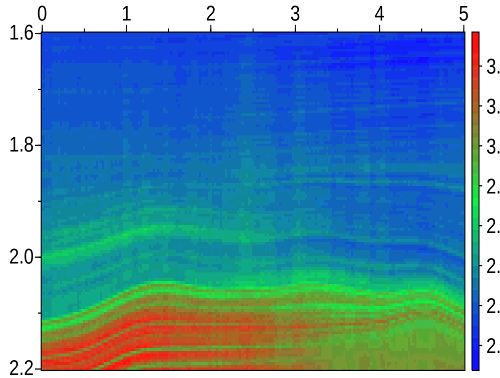
Initial P vel. (km/s)



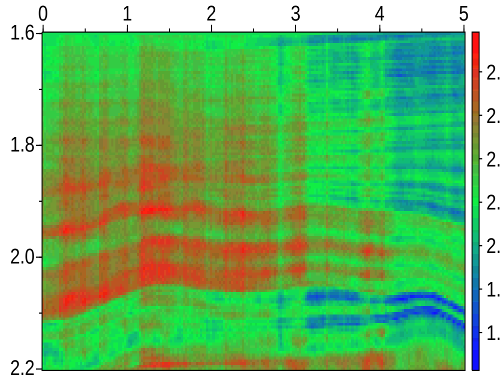
Initial density (g/cm3)



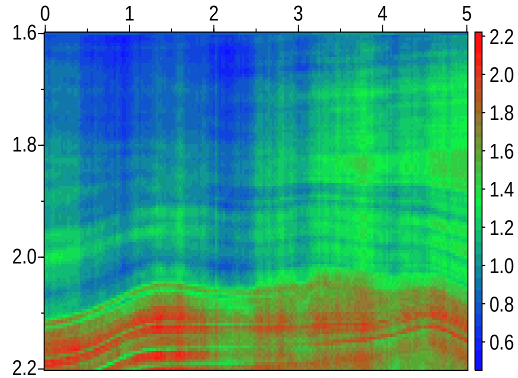
Initial S vel. (km/s)



Inverted P vel. (km/s)



Inverted density (g/cm3)



Inverted S vel. (km/s)

Concluding remarks and further directions

- **Its is possible to greatly improve waveform match by relatively small changes to the well log models.**

- **The solution is quite sensitive to the source wavelet (assumed known) and the *a priori* model.**

- **Possible future improvements in the method:**
  - **to constrain the model space by use of rock physics relations**
  - **include PS converted waves (for ocean bottom recording)**
  - **include anisotropy (VTI)**
  - **include attenuation**
  - **allow source wavelet to depend on incidence angle**
  - **estimate source wavelet**
  - **include multiples (when applied to pre-stack data)**

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