GEOF 270

Applied Seismology

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Introduction

The course gives an introduction to practical methods in seismology: seismic instruments, seismic source parameters and their determination, fault plane solutions, seismic waves and the Earth's interior.

A term paper counts for 30 %

The time of teaching is set up in agreement with the students.

Teaching material

Stein and Wysession, An introduction to seismology, earthquakes, and earth structure, Blackwell Publishing.

Havskov and Alguacil, Instrumentation in Earthquake Seismology, Springer, chapter 1 and 3 (included below).

The pensum and order of teaching is:

Introduction
p 1-28

Instruments
p 398-412, p 2-16 and 20-21 in this document (Havskov and Alguacil)

Phases and rays
p 162-176

The earthquake source
p 215-251

Source parameters
p 263-282

Inverse problems, location and moment tensor
p 415-434

Attenuation
p 185-198

Chapter 1

Introduction

Seismology would be a very different science without instruments. The real big advances in seismology happened from around 1900 and onwards and was mainly due to advancement in making more sensitive seismographs and devising timing systems, so that earthquakes could be located. Later, the importance accurate measurement of the true ground motion became evident for studying seismic wave attenuation, and the Richter magnitude scale depends on being able to calculate the ground displacement from our recorded seismogram (Figure 1.1).

The ability to do earthquake location and calculate magnitude immediately brings us into two basic requirement of instrumentation: keeping accurate time and determining the frequency dependent relation between the measurement and the real ground motion.

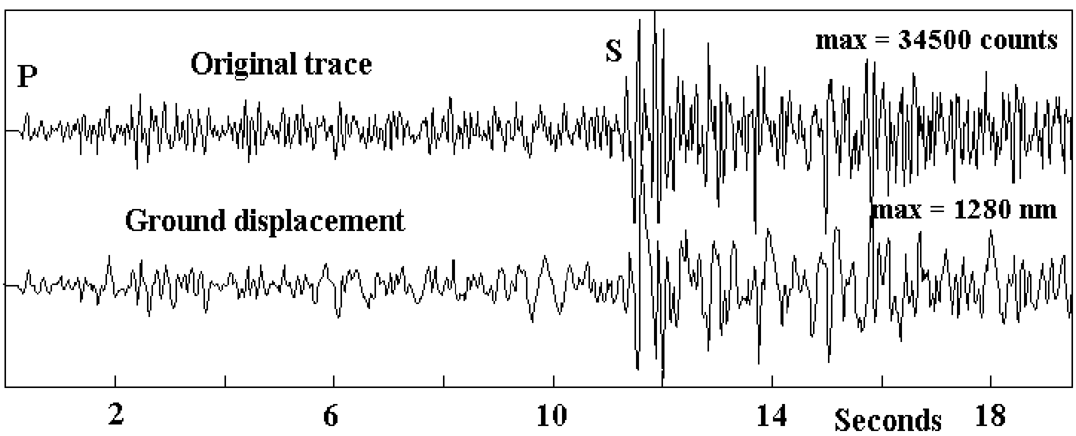

Figure 1.1. The top trace shows the original digitally recorded signal from a magnitude 3 earthquake recorded at a distance of 120 km. The maximum amplitude is just a number (called counts). The bottom trace shows the signal converted to true ground displacement in nm from which the magnitude can be calculated. The distance to the earthquake is proportional to the arrival time difference between the S-wave and P-wave, so having 3 stations makes it possible to locate the earthquake. The seismometer is a 1 Hz sensor with velocity output.

Seismologists tend to take their data for granted, hoping that the black boxes of seismographs and processing software will take care of all the nasty problems to just give the correct ground motion (Figure 1.1), much like driving a car and not worrying too much of how it works. But cars stop or malfunction and so seismographs, so a basic understanding of seismic instrumentation is essential, even for the seismologist who is never going to turn a knob on an instrument. Like getting some data with a few instrumental constants and trying to figure out how it can be used to calculate the true ground motion.

Instrumentation is not just a topic for seismologists, since most equipment is in fact installed and maintained by non-seismologists, so this group of professionals has just as much a need for information on instrumentation.

There has been numerous publications on instrumental topics in seismology, with very special emphasis on particular topics, but few general textbooks. A general overview was made by Lee and Stewart (1981), which, on the instrumental side, mainly dealt with micro- earthquake networks. The old Manual of Seismological Observatory Practice (MSOP) (Wilmore, 1979) dealt with all the classical analog seismographs, but is partly outdated now. The New Manual of Seismological Observatory Practice (NMSOP, Borman, 2002)) is the most up to date book on seismic instruments and one of the authors of this book (JH) has also participated in NMSOP. So why another book? The NMSOP deals with a lot of topics in addition to seismic instruments, and we felt there would be a need for a book which further expands on the instrumental topics, more than was possible within NMSOP, and put it all together in one volume. The intention with this book is that it should be a practical tool with only the amount of theory needed to understand the basic principles. However, without NMSOP it would have been difficult to write this book.

So what are the main topics of instrumental seismology? It all starts with being able to measure the ground motion, and this is the most important topic. Then follows recording and/or transmission to a central site.

We usually talk about ground displacement (formally measured in m) since this is what seismologists like to use. Like a fault has a displacement of 2 m during an earthquake, we can talk of a ground displacement of 2 m, although we normally are measuring much smaller motions. On the other hand, engineers, seem to think that acceleration (m/s^2) is the most natural unit, since it is directly related to force and the peak ground acceleration is an often quoted measure.

The range of amplitudes is very large. The natural background noise, highly frequency dependent, sets the limit for the smallest amplitudes we can measure, which is typically 1 nm displacement at 1 Hz (Chapter 2), while the largest displacement is in the order of 1 m. This is a dynamic range of 10^9. The band of frequencies we are interested in, also has large range, from 10^{-5} to 1000 Hz (Table 1). The challenge is therefore to construct seismic instruments, both sensors and recorders, which cover at least part of this large frequency and dynamic range.

Frequency (Hz)	Type of measurements
0.00001-0.0001	Earth tides
0.0001-0.001	Earth free oscillations, earthquakes
0.001-0.01	Surface waves, earthquakes
0.01-0.1	Surface waves, P and S waves, earthquakes with M > 6
0.1-10	P and S waves, earthquakes with M> 2
10-1000	P and S waves, earthquakes, M< 2

Table 1.1 Typical frequencies generated by different seismic sources

Earlier, analog instrument were usually made to record one type of ground motion like velocity. Traditionally, seismologists prefer recording weak motion displacement or velocity, for easy interpretation of seismic phases, while engineers use strong motion acceleration, whose peak values are directly related to structures seismic load Today it makes less of a difference, since due to advancement in sensor and recording systems, the weak motion instruments can measure rather strong motions and the strong motion sensors are almost as sensitive as the weak motion sensors. The digital recording furthermore makes it easy to convert from acceleration to velocity etc., see Figure 1.2

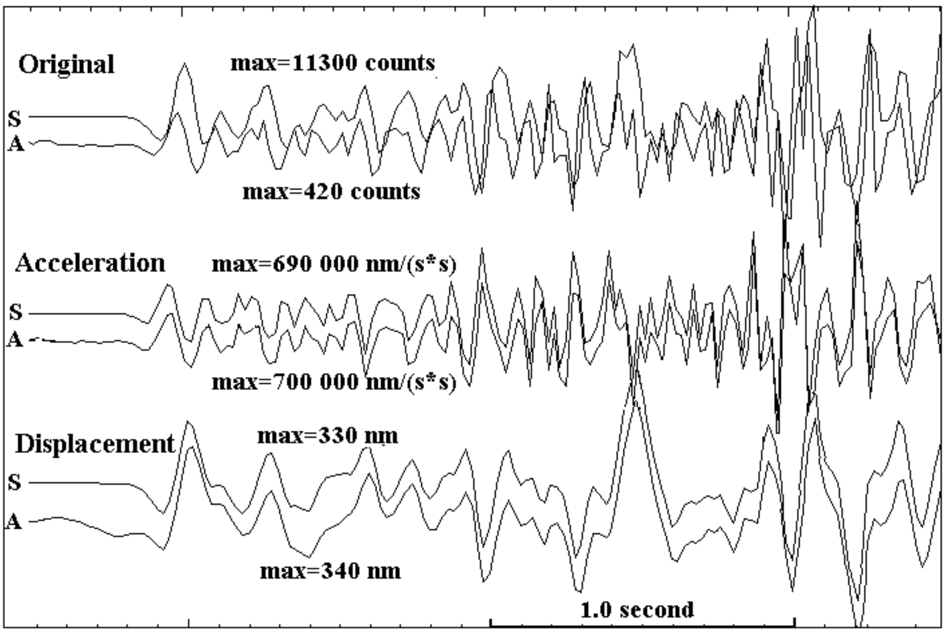

Figure 1.2 Acceleration and displacement. The seismogram in the figure is the first few seconds of the P-wave of the signal seen in Figure 1.1. On the site there is also an accelerometer installed (A) next to the seismometer (S). The top traces show the original records in counts. The signal from the seismometer is similar to the accelerometer signal, but of higher frequency and the amplitudes are different. The middle traces show the two signals converted to accelerations and the bottom traces, converted to displacement (frequency band 1-20 Hz). The signals are now very similar and of the same amplitude. This example clearly demonstrates that, with modern instrument and processing techniques, we can use both accelerometers and (velocity-sensitive) seismometers and get the same result.

Sensor

Since the measurements are done in a moving reference frame (the earth's surface), almost all seismic sensors are based on the inertia of a suspended mass, which will tend to remain stationary in response to external motion. The relative motion between the suspended mass and the ground will then be a function of the ground's motion (Figure 1.3). The swinging system will have a resonance frequency f_0

$$f_0 = \frac{1}{2\pi}\sqrt{k/m} \qquad (1.1)$$

where *k* is the spring constant and *m* the mass. If the ground displacement frequency is near the resonance frequency, we get a larger relative motion (depending on damping) and, as it turns out, below the resonance frequency, the relative displacement, due to ground displacement, decreases (Chapter 2).

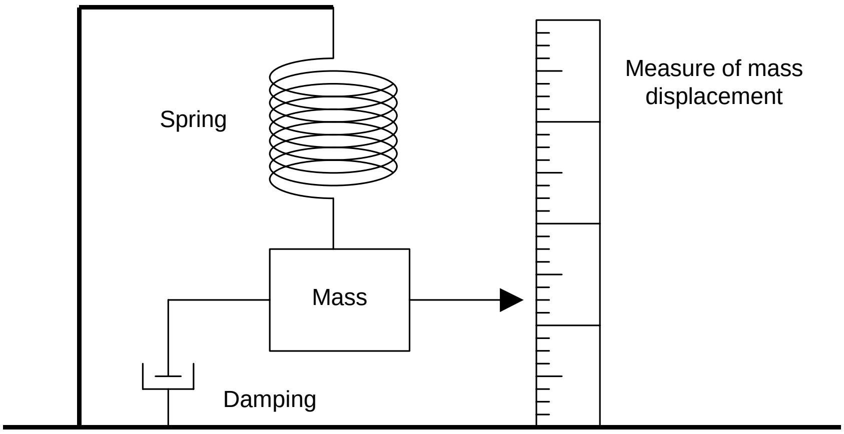

Figure 1.3 Principle behind the inertial seismometer. The damping of the motion can be mechanical, but is usually electro-magnetic.

The sensor is moving with the ground and there is not any fixed undisturbed reference available. So displacement or velocity cannot be measured directly. According to the inertia principle, we can only observe the motion if it has a non zero acceleration. So if we put a seismic sensor in a train, we can only measure when the train is accelerating or braking. As seismologist, we want to measure displacement, but this is not possible to do directly. We have to measure the ground acceleration and integrate twice. Displacement at very low frequencies produce very low accelerations since

$$\ddot{u} \propto f^2 u \quad (1.2)$$

where *u* is the ground displacement and *f* the frequency. It is therefore understandable why it is so difficult to produce seismometers that are sensitive to low frequency motion. So, while it is quite straight forward to make a sensor which records equally well a given acceleration level at all frequencies, even DC, it is much more difficult to measure slow displacements. Much of the advances in recent years in seismic instruments has been the ability to build sensors with a better low frequency sensitivity.

Earlier, sensors sensitive to low frequency were made with sophisticated mechanical systems, which by all sorts of tricks were able to have a low resonance frequency. It was however not possible to make sensors with a stable resonance frequency much lower than 0.03 Hz. Today, purely mechanical sensors are only constructed to have resonance frequencies down to about 1.0 Hz (short period sensors), while sensors that can measure lower frequencies are based on the Force Balance Principle (FBA) of measuring acceleration directly.

The sensor mass displacement is linearly proportional to the external acceleration, even at zero frequency since this just corresponds to a permanent change in the external force. As we shall see in Chapter 2, this linearity does no hold for frequencies above the resonance frequency, where the mass displacement due to ground acceleration will drop proportional to frequency squared. But since it is easy to construct a swinging system with a high resonance frequency (small mass and/or stiff spring, equation (1.1)), we can easily make an accelerometer by just measuring the mass displacement of a spring suspended mass for frequencies below the resonance frequency. So we have our mechanical accelerometer, at least in theory. The problem is now how to measure this mass displacement, particularly at low frequencies, where the acceleration is small. The popular velocity transducer (gives out a voltage proportional to the mass velocity relative to the frame) is not a good choice since the velocity goes down with frequency and is zero at zero frequency, so even if there is a permanent acceleration, we cannot measure it. Special mass displacement transducers have been introduced, but they are hard, if not impossible, to make accurate for the large dynamic range we need in seismology. However, for a small displacement, they can be made very sensitive and accurate. This is what is used in the so called Force Balance Principle (Figure 1.4).

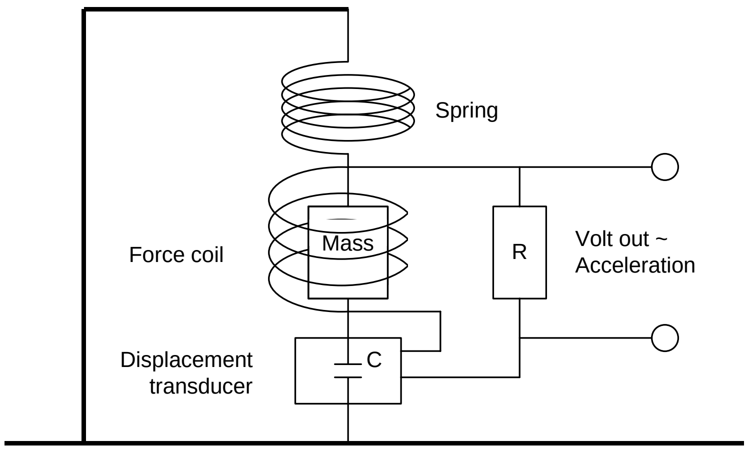

Figure 1.4 Simplified principle behind Force Balanced Accelerometer. The displacement transducer normally uses a capacitor *C*, whose capacitance varies with the displacement of the mass. A current, proportional to the displacement transducer output, will force the mass to remain stationary relative to the frame.

The Force Balanced Accelerometer (FBA) has a feedback coil, which can exert a force equal and opposite to the inertia force due to the largest acceleration we want to measure. The displacement transducer sends a current through this force coil through a resistor *R* in a negative feedback loop. The polarity of the current is such that it opposes any motion of the mass, and it will try to prevent the mass from moving at all with respect to the frame. A small permanent acceleration on the mass will therefore result in a small permanent current and a large acceleration will need a large current. The current is in fact linearly proportional to the ground acceleration, so a direct measure of acceleration is given by the voltage over the resistor.

The FBA principle is now the heart of nearly all modern strong motion and broad band sensors (sensors recording in a large frequency band like 0.01 to 50 Hz). By connecting an integrating circuit, either in the feedback loop, or after, the sensor can give out a voltage linearly proportional to velocity. However, due to the inertial principle, there must be a low frequency limit (velocity is zero at zero frequency!) and this limit is set by the mechanical-electrical qualities of the sensor. Currently, the best broad band sensors have a limit of about 0.0025 Hz, much lower than it was ever possible with purely mechanical sensors. The FBA principle also has the advantage of making linear sensors, with a high dynamic range since the mass almost does not move. Currently, the best sensors have a dynamic range of 10^6 - 10^8.

So, in summary, mechanical sensors are the most used when the natural frequency is above 1 Hz, while the FBA type sensors are used for accelerometers and broad band sensors. This means that a lot of sensors now are ‘black boxes’ of highly complex mechanical and electronic components of which the user can do very little. However, as we shall see in the calibration sections, Chapter 10, it is still possible to do a series of simple tests to check the functionality of the instrument.

Recorder

The next challenge is to be able to actually record the signals from the seismometer and not loose quality. Analog recording is still used (Figure 1.6) since it gives a very fast and easy overview of the seismicity. However, the dynamic range is very limited (at most 200 in contrast to 10^8). Analog recorders are very expensive to buy and operate, so they are rarely installed anymore. In addition, few seismic recorders give out an analog signal required of the recorders so an additional digital to analog converter must be installed.

The development of high dynamic range and broad band sensors would not have improved seismology a lot unless the recording technique had followed. Fortunately, the last 20 years have seen a fast development in digital recording, which almost has been able keep up with the improvement in sensor development. Converting the analog signal to a digital signal means converting the continuously varying signal to discrete samples of the analog signal, see Figure 1.5.

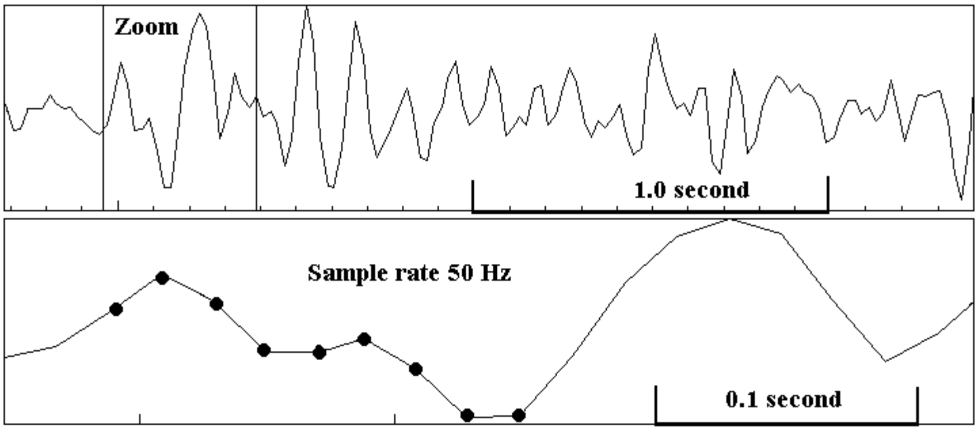

Figure 1.5 Example of a digitized earthquake signal. The top trace shows the signal with a low time resolution and the bottom trace, the zoom window with a high resolution. In the zoom window, the discrete samples are clearly seen. The sampled points are indicated with black dots

The digitizer will thus convert a voltage to a number, just like we do with a digital voltmeter. The difference is that we have to do it many times a second on a varying amplitude signal. From the figure, we se that we are loosing information between each two samples, so that intermediate changes cannot be seen after digitization. We can also see two important aspects of the digitizer: The sample interval (step in time direction) and the resolution (step in amplitude direction), which corresponds to one step in the numerical output from the digitizer. If more high frequency content is needed, it is just a question of increasing sample rate (except that the dynamic range usually deteriorates with a higher sample rate) and we have to sample at least at twice the rate of the frequency of the signal we want to record. The step in the amplitude direction depends on the digitizer resolution and is the smallest step it can resolve. The best digitizers can usually resolve 100 nV which then ideally should correspond to the number 1. So, what about the dynamic range? Assuming that the number 1 out of the digitizer corresponds to the real input signal (and not digitizer noise), then the largest number corresponds to the dynamic range. Digitizers are usually classified as 12 bit, 16 bit or 24 bit which has to do with the number of discrete values the digitizer uses. A 12 bit converter has 2^{12} levels and a 24 bit converter 2^{24} levels or $\pm 2^{11}$ and $\pm 2^{23}$ respectively. This corresponds to dynamic ranges of 2048 and 8.4 10^6 respectively. Now most digitizers are based on 24 bit analog-to-digital converters (ADC), however few have a true 24 bit performance, since shorting the input gives out counts significantly different from zero. So the digitizers do not quite match the best sensors, although there a few digitizers approaching and above the true 24 bit performance. The standard quality 24 digitizers usually only reach 21-22 bit dynamic range.

Digitizer and recorder are often one unit although they physically are two separate parts. Recording is now mostly based on hard disks, although some units use solid state memory. The trend is to use more and more powerful computers. While many units on the market today use microcontrollers in order to save power, the trend is to use complete single board PC's due to the low power consumption of these new generation PC's. The advantage of using real PC's with 'normal' operating systems is that all standard software, e.g. communication, can be used instead of the manufacturer special fix for his special recorder.

Since large hard disks can be used for recording, recorder memory is no longer a problem and continuous recording is now easy to do. However, nearly all recorders also have a facility for additionally storing only the time segment containing real events and the system is said to have a trigger. This means that a program continuously monitors the incoming signal levels to decide when a real event occur.

Stations and networks

So now we have a complete system and it is just putting it in the field, or isn't it ? If we are going to be able to record a signal level of 1 nm, we cannot just put the sensor anywhere. Apart from the earth's natural background noise, which well might be larger than 1 nm at 1 Hz (Chapter 3), we must take other noise disturbances into account like traffic, wind induced noise etc. So before installing a station, a noise survey must be made (Chapter 7). In addition, broad band sensors require special attention due to the large sensibility to low frequency disturbances created mainly by temperature induced tilt. So while it might be simple and cheap to install a short period seismometer, a broad band installation might be time consuming and expensive. In some cases, it might even be necessary to install it in a borehole to get away from the noise near the surface. Another method of improving the signal to noise ratio, is to use a seismic array (Chapter 9).

One station does not make a network, which is what we normally want for locating earthquakes. Setting up a network is mainly a question about communication. This field is in rapid development and will probably completely change our concept of how to make seismic networks. Earlier, a network was often a tightly linked system like the classical microearthquake network of analog radio telemetered stations a few hundred kilometers away sending analog data from the field stations to a central digital recorder (Figure 1.6). While these types of networks still are being built using digital transmission, the trend is for field stations to become independent computer nodes which, by software, can be linked together in *virtual seismic networks*, so called to distinguish them from the more classical physical networks. So, in the virtual network, the communication system is a design part separated from the seismic stations themselves. The communication can be based on radio, satellite or public data communication channels.

Since data from many different station are used together, all signals must be accurately timed. Today, this is not a problem since the GPS (satellite based Global Positioning System) system delivers very accurate time and GPS receivers are cheaply available.

Figure 1.6 The central recording site at the Andalusia Seismic Network. The network consists of analog seismic station sending analog signals to the central recording site. Here the signals are digitized using the PC and events triggering the system are stored on disk. In addition, data are recorded in analog form on drum recorders (helicorder). In this photo we see an example where a seismic swarm is recorded making it possible to get an easy overview of the recent activity.

Arrays

Ground motion measured by a seismic station is a sample of one point of a spatial wavefield that propagates in time. A set of stations may be deployed in an area in such a way that the whole wavefield in this area is sampled and the data processing of all waveforms is performed jointly. We call this set of seismic stations a *seismic array*. Linear arrays have been used for seismic surveys for long, but the term is now applied to 2-D or even 3-D arrangements for studying the seismic source, wave propagation, shallow structure, etc.

Use of arrays makes it possible to "direct" the sensitivity of the network to a source point for instance, with an improved signal-to-noise ratio (SNR) over the individual stations. Its applications have been extended in recent years from global seismology and nuclear test ban verification to local and regional seismology, including the seismo-volcanic sources, to be used as a powerful tool for seismologists.

Instrument correction and calibration

We are now recording a lot of nice signals and some seismologist is bound to start using it; after all, that is why we set up seismic stations. The signal will then be displayed on screen and the maximum amplitude will be e.g. 23838. That is not much help if we want to know what the true ground motion is, so we now have figure out what the relation is between the true ground motion and the numbers out of the computer. This involves two steps: (1) Getting the true parameters for each unit of the sensor and the recorder, which might involve doing laboratory work, but at least consulting manufacturer manuals and (2) Being able to calculate the systems response function *T(f)* which simply is defined as

$$T(f) = \frac{Z(f)}{U(f)} \qquad (1.3)$$

where f is the frequency, Z in the recorded signal and U the true ground signal. Knowing the response function, we can then calculate the true ground signal. Unfortunately, the story does not end here since the function *T(f)* can be specified in many different ways depending on data format and seismic processing systems, and it is not uncommon to get different results. So working with response information can be quite confusing. Figures 1.1 and 1.2 show examples of signals corrected for instrument response.

By now, the seismologists should have complete correct data available for a well calibrated seismic station installed at an ideal location with a perfect operating seismic network, if all recommendations in this book have been followed!! But it is not time to relax yet. When analyzing the data, it might become evident that not all is well. The hope is that by knowing more about seismic instruments, problems should be spotted soon, and it should be relatively easy to find out where the problem is. Help is then at hand to do some calibrations and tests. Repairing the equipment is another story and best left to the true experts.

Chapter 3

Seismic noise

Recorded seismic signals always contain noise and it is important to be aware of both the source of the noise and how to measure it. Noise can have two origins: Noise generated in the instrumentation and 'real' seismic noise from earth vibrations. Normally, the instrument noise is well below the seismic noise although most sensors will have some frequency band where the instrumental noise is dominating (e.g. an accelerometer at low frequencies). The instrumental noise is dealt with in more detail in Chapter 2 on seismic sensors. So from now on in this section, it is assumed that noise is seismic noise.

3.1 Observation of noise

All seismograms show some kind of noise when the gain is turned up and at most places in the world, harmonic-like noise (called microseismic noise) in the 0.1 - 1.0 Hz band is observed in the raw seismogram, unless obscured by a high local noise level (Figure 3.1). From Figure 3.1, it is also seen, that although the microseismic noise dominates (see noise sources later), there is also significant seismic noise in other frequency bands. So obviously, the noise level must be specified at different frequencies.

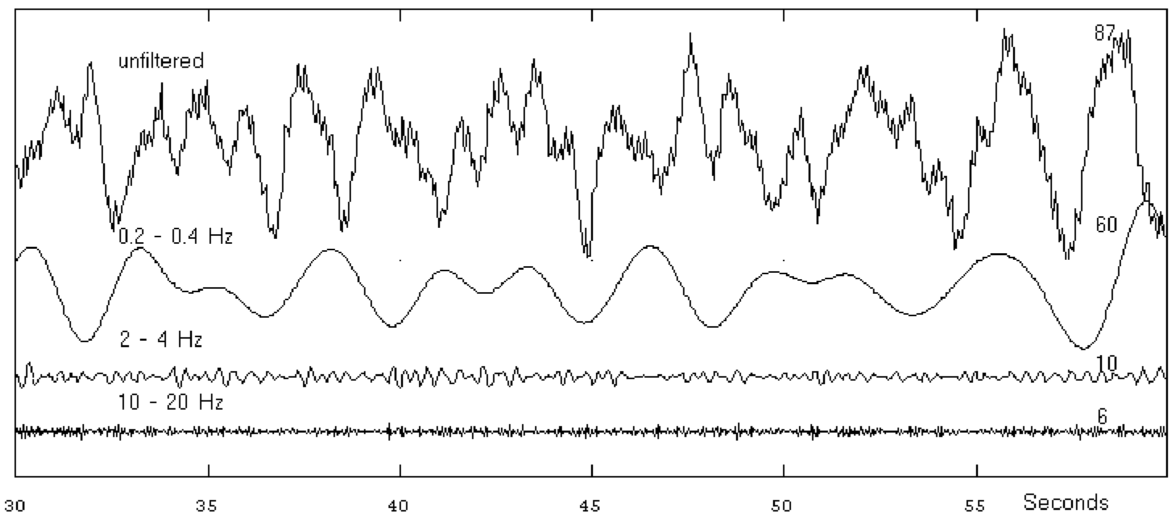

Figure 3.1 Seismic noise in different filter bands at station MOL in the Norwegian national seismic network The short period station (1 Hz) is situated about 40 km from the North Sea and the unfiltered trace clearly shows the high level of low frequency noise (0.3-0.5 Hz) generated by the sea. All traces are plotted with the same scale and the numbers to the right above the traces are the maximum amplitude in counts.

Intuitively, the simplest way should be to measure the earth displacement in different frequency bands and plot the amplitude as a function of frequency or period. This was in fact the way it was done before the use of digital recordings and an example of measurements from the old manual of Seismological Observatory Practice (Willmore, 1979)) is shown in Figure 3.3.

The use of filtering and measurements in the time domain presents 2 problems:

(1) Bandwidth used is an arbitrary choice, (2) Getting average values over long time intervals. Both of these problems are solved by presenting the noise in the spectral domain, (see Chapter 6 for more on spectral analysis). Figure 3.2 illustrates the problem of the filter bandwidth. The figure shows the same signal filtered with an increasingly narrow filter. The amplitude of the signal decreases as the filter becomes narrower, since less and less energy gets into the filter band. In the example in Figure 3.2, the maximum amplitude at around 1 Hz varies from 23 nm to 6 nm depending on filter width. This decrease in amplitude is mainly due to making the filter more narrow. However, for the widest frequency band (0.6 –1.7 Hz), relatively more low frequency energy is also present in the original signal. Comparing to Figure 3.3, the noise level can be considered worse or better than average depending on which filter band is used. So in order to do measurements in time domain, the noise values can only be compared if the same bandwidth of the filter is used.

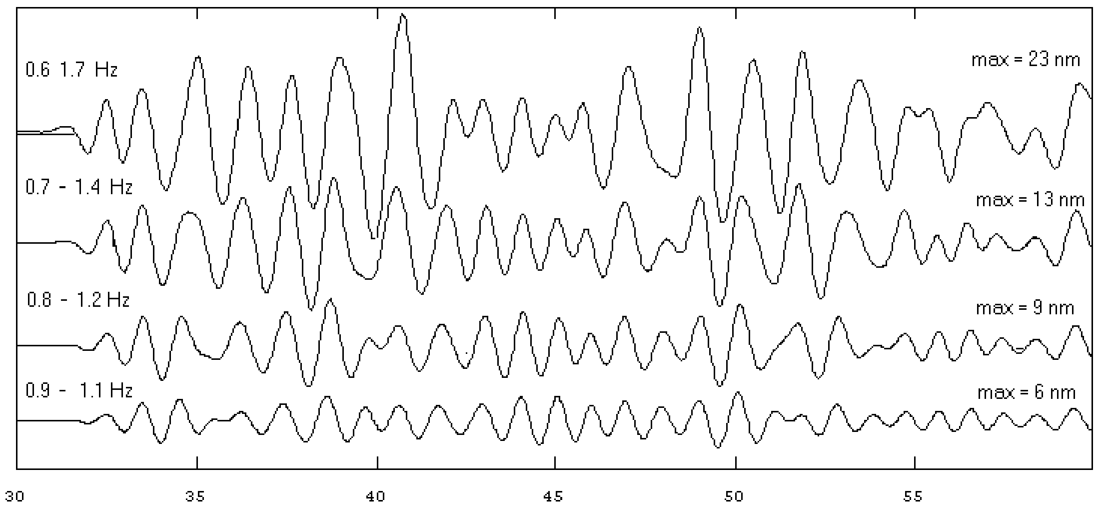

Figure 3.2 The signal from Figure 3.1 band pass filtered with different filter widths. The signals have been corrected for instrument response to show displacement. The maximum amplitude in nm is shown to the right on top of the traces.

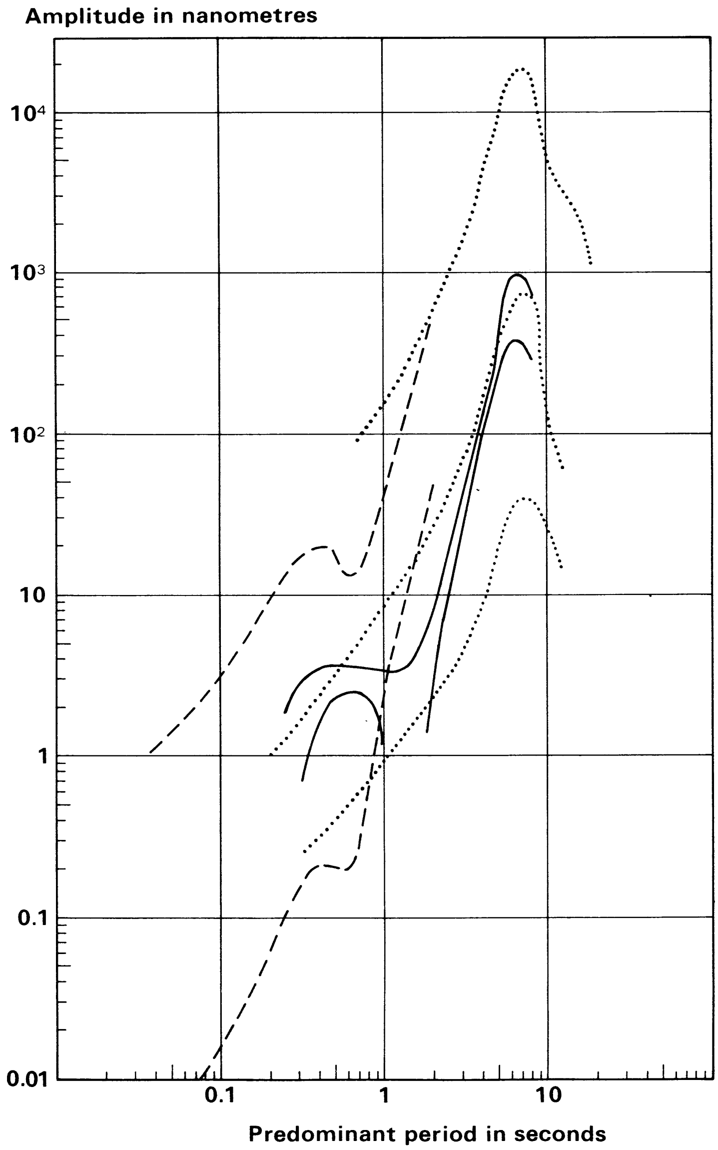

Figure 3.3 Noise curves in rural environment. The 3 dotted lines correspond to the maximum, mean and minimum levels published by Brune and Oliver (1959), the dashed lines give two extreme examples observed in the US and the full line curves gives the limits of fluctuation of seismic noise at a European station on bedrock in a populated area 15 km away from heavy traffic (from Wilmore, 1979).

3.2 Noise spectra

With digital data, it is possible to make spectral analysis, and thereby easily get the noise level at all frequencies in one simple operation. It has become the convention to represent the noise spectra as the noise power density acceleration spectrum $P_a(\omega)$. It has become common to represent the spectrum in units of dB referred to 1 $(m/s^2)^2$/Hz. Noise Level is thus calculated as

$$Noise\ Level = 10 \log [P_a(\omega)/(m/s^2)^2/Hz] \quad (3.1)$$

Figure 3.4 shows the new global high (NHNM) and low noise models NLNM) (Peterson, 1993) and an example of noise spectra at a seismic station. The curves represent upper and lower bounds of a cumulative compilation of representative ground acceleration power spectral densities determined for noisy and quiet periods at 75 worldwide distributed digital stations. These so-called Peterson curves have become the standard, by which the noise level at seismic stations is evaluated. *A power density spectrum can be defined in different ways and it is important that the definition used is identical to one used originally (Figure 3.4) in order to compare the spectra. Chapter 6 gives the exact definition.*

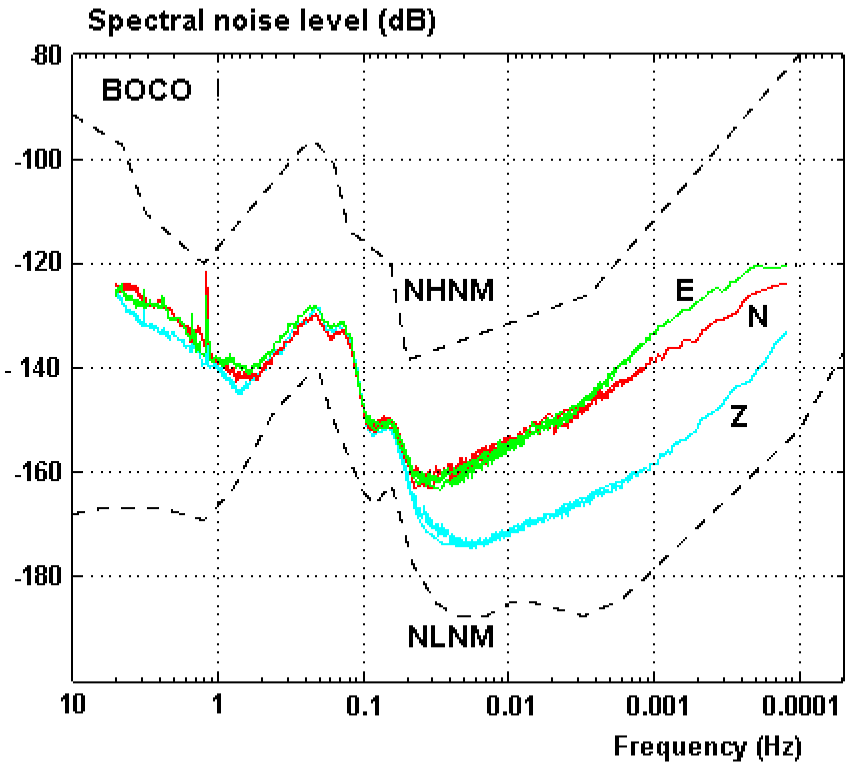

Figure 3.4 The Peterson noise curves and noise spectral level for the IRIS station BOCO. The noise level is in dB relative to 1 $(ms^{-2})^2$/Hz. The Peterson high and low noise models are shown with dashed lines. The noise spectra are shown for all 3 components. Note the lower noise level for the vertical (Z) component. Figure modified from IRIS station book (note that x - axis represents decreasing frequency).

3.3 Relating power spectra to amplitude measurement

The Peterson noise curves and the way of representing them have standardized the way of representing seismic noise. However, looking at such a curve, it is difficult to relate them to something physical as seen in Figures 3.1, 3.2 and 3.3 and a standard question is often: So what is the physical meaning of the Peterson curve? The old noise curve in Figure 3.3 can be directly related to the seismogram in Figure 3.2, while the Peterson curve cannot since one is a frequency domain measure and the other a time domain measure. However, as will be shown below, under certain conditions it is actually possible to go from one to the other (the following text largely follows NMSOP (Borman, 2002)).

The problem is how to relate a spectral amplitude at a given frequency to a time domain amplitude in a given frequency band. The root mean squared amplitude, a_{RMS} of a signal in the time interval *0-T*, is defined as

$$a_{RMS}^2 = \frac{1}{T}\int_0^T a(t)^2\,dt \qquad (3.2)$$

The average power of the signal in the time interval is then equal to a_{RMS}^2. The average power can also be calculated (Parseval´s Theorem) from the power density spectrum as

$$a_{RMS}^2 = \int_{f_1}^{f_2} P(\omega)df \approx P\cdot(f_2 - f_1) \qquad (3.3)$$

under the assumption that the power spectrum is nearly a constant *P* in the frequency range f_1 to f_2, which is not unreasonable if the filter is narrow. In the general case, *P* would represent the average value of *P(ω)* in this frequency band. It is important that *P(ω)* is the normalized power spectral density as defined in Chapter 6 and that the power represent the total contribution at *ω* from both positive and negative frequencies. If, as is the usual practice, the power spectrum is calculated using only the positive frequencies, a^2_{RMS} will have to be calculated as $2P(f_2\text{-}f_1)$. **The power values given by the New Global Noise Model by Peterson (1993) (see Fig. 4.7) do already contain this factor of two, or said in other words, represent the total power.**

Under these assumptions, we then have a relationship between the power spectral density and the RMS amplitude within a narrow frequency band:

$$a_{RMS} = \sqrt{P\cdot(f_2 - f_1)} \qquad (3.4)$$

There is thus a simple way of relating the power spectral density to amplitudes, as seen on a seismogram, however note that relation (3.4) gives the RMS amplitude. There is statistically a 95% probability that the instantaneous peak amplitude of a random wavelet with Gaussian amplitude distribution lie within a range of $2a_{RMS}$. Peterson (1993) showed that both broadband and long period noise amplitudes closely follow a Gaussian probability distribution. In the case of narrowband-filtered envelopes, the average peak amplitudes are 1.25 a_{RMS} (Borman, 2002). From measurement of noise using a narrowband filtered VBB data, values of 1.19 to 1.28 were found (Peterson, 1993). Thus, in order to get the true average peak amplitude on the seismogram, a factor of about 1.25 can be used. (NB: For a pure sine wave, $a = a_{RMS}\sqrt{2}$, not so very different). We can now set up the relation between the power spectral values and the average peak amplitudes

$$a = a_{RMS}\cdot 1.25 = 1.25\sqrt{P\cdot(f_2 - f_1)} \qquad (3.5)$$

The frequency band depends on instrument (mainly if analog), while for digital data the user can select the filter. A common way of specifying filter bands is to use the term octave filter. An *n*-octave filter has filter limits

$$\frac{f_2}{f_1} = 2^n \tag{3.6}$$

For example, a half-octave filter has the limits f_1 and $f_2 = f_1^{1/2}$ (e.g. 1-1.42 Hz). Many of the classical analog seismographs have band widths of 1-3 octaves and digital seismographs might have a bandwidth of 6-12 octaves. However, the signal bandwidth of many dominating components of seismic background noise might be less then 1 octave. Thus, the frequency of measurement is really a frequency range. However, for practical reason, the average frequency will be used to represent the measurement. For the average frequency, the geometric center frequency f_0 must be used

$$f_0 = \sqrt{f_1 f_2} = \sqrt{f_1 f_1 \cdot 2^n} = f_1 \cdot 2^{\frac{n}{2}} \tag{3.7}$$

and

$$f_1 = f_0 2^{-\frac{n}{2}} \quad \text{and} \quad f_2 = f_0 2^{\frac{n}{2}} \tag{3.8}$$

For narrow filters, the geometric center frequency is almost the same as the average frequency. The filters used in Figure 3.2 have average frequencies of 1.2, 1.1, 1.0 and 1.0 Hz while the geometric center frequencies are all 1.0. For the filter 10-20 Hz, the geometric and average frequencies are 14 and 15 Hz respectively.

In comparing time and frequency domain signals, there was no mention of units since this has no importance on the relations. However, the Peterson curves are in acceleration, so unless the time domain signal is in acceleration too, the power spectrum must first be transformed to acceleration. The most common unit for the original seismogram is velocity, sometimes acceleration and rarely displacement. If the power spectra of acceleration, velocity and displacement are called P_a, P_v, and P_d respectively, the relations are: .

$$P_v(\omega) = P_d(\omega) \cdot \omega^2 \tag{3.9}$$

$$P_a(\omega) = P_v(\omega) \cdot \omega^2 = P_d(\omega) \cdot \omega^4 \tag{3.10}$$

Now some examples. Figure 3.2 has a peak amplitude of 13 nm for the frequency band 0.7.-1.4 Hz. This can be converted to P_d

$$P_d = (13 * 10^{-9}/1.25)^2/(1.4 - 0.7) = 1.55 \cdot 10^{-16}\ m^2/Hz \tag{3.11}$$

The center frequency is $\sqrt{0.7 \times 1.4} = 1.0$ Hz and the acceleration power density is

$$P_a = \quad 1.55 \cdot 10^{-16} \cdot 16 \cdot \pi^4 \cdot 1^4 = 2.4 \cdot 10^{-13}\ (m/s^2)^2/Hz \tag{3.12}$$

or –126dB relative to 1 $(m/s^2)^2$/Hz. Figure 3.5, shows the complete noise power density spectrum for the time window used in Figures 3.1 and 3.2. It is seen that the level –126 dB is high compared to the spectral level which is around –136 dB. How can that be explained ? We have to remember that (3.5) is based on the assumption that the amplitude is the *average peak amplitude,* while what has been used is the largest amplitude in the window. If the average peak amplitude is 3 times smaller than the maximum amplitude, the level would be corrected by $-10 \log(3^2) = -9.5$ dB and the time domain and frequency domain measures would be similar.

This discussion should demonstrate, that by far the most objective way to report the noise level at a given site is to use the power density spectrum although is nice to be able to relate it, at least approximately, to some amplitude measure.

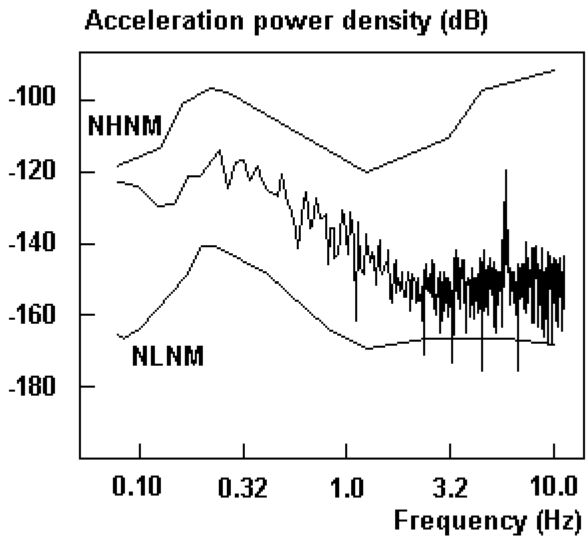

Figure 3.5 Noise power density spectrum of the raw signal seen in Figure 3.1, top trace. The acceleration power is in dB relative to 1 $(m/s^2)^2$/Hz. The limits of the Peterson noise model is indicated (NHNM and NLNM) . The spectrum has not been smoothed.

The spectrum in Figure 3.5 show a relatively high noise level at lower frequencies relative to high frequencies. This is not surprising considering the general high microseismic noise level along the Norwegian West coast. The noise level above 3 Hz is quite low since the station is on granite in a rural area (20 m from nearest house).

The noise spectral level can be calculated for all 4 cases in Figure 3.2, see Table 3.1. It is seen, that although the amplitudes were quite different in the 4 filter bands, power spectral levels are nearly equal except for the filter band 0.6 – 1.7 Hz where the amplitudes in the relatively wide filter is influenced by the stronger background noise at lower frequencies.

Filter bands (Hz)	Amplitude(nm)	Noise power level (db)
0.6 –1.7	23	- 123
0.7 – 1.4	13	- 126
0.8 – 1.2	9	- 127
0.9 – 1.1	6	- 127

Table 3.1 Maximum amplitude in the different filter bands and corresponding noise levels**.**

From Figure 3.4 or 3.5, we see that the NLNM has a level of –166 dB at 1 Hz. Assuming a 2 octave filter and using (3.5) and (3.8), the corresponding average peak displacement is 0.3 nm. From Figure 3.3 it is seen that the lowest displacement is 1 nm at 1 Hz so there is a reasonable agreement considering the uncertainty of whether RMS values or average peak values have been used. At 10 Hz, the NLNM gives 0.01nm which also agrees well with the values on Figure 3.3. So rule of thumb (and easy to remember) is: *A peak displacement of 1 nm at 1 Hz means a good site in terms of ambient noise.*

One point to be underlined is that noise is of random nature (any "predictable" disturbance is not noise as defined here!). The power spectrum estimated by the simple Fourier transform of a sample window of raw data is also random. The standard error of such an estimate is very high (see, e.g. Blackman and Tukey, 1966; Press et al., 1995). Actually, when we express the noise level as the power spectrum density or RMS within a certain frequency band, we are implicitly assuming that noise is a stationary process, which means that its statistical characteristics are not time-dependent or at least vary slowly enough to be considered constant within a certain time interval.

Therefore, if a reliable estimation of the noise level, at a given site, is needed, an average of power density estimation on several overlapping sample time-windows or some spectral smoothing will decrease the estimate variance (e.g. Blackman and Tukey, 1966).

3.4 Origin of seismic noise

Man made noise: Often referred to as "cultural" noise, it originates from traffic and machinery, has high frequencies (>2-4 Hz) and die out rather quickly (m to km), when moving away from the noise source. It propagates mainly as high-frequency surface waves, which attenuate fast with distance and decrease in amplitude strongly with depth, so it may become almost negligible in boreholes, deep caves or tunnel sites. This kind of noise usually has a large difference between day and night and can have characteristic frequencies depending on the source of the disturbance. The noise level can be very high.

Wind noise: Wind will make any object move so it will always generate ground noise. This noise is usually high frequency like man made noise, however large swinging objects like masts and towers can generate lower frequency signals. Trees also transmit wind vibrations to ground and therefore seismic stations should be installed away from them. In general, wind turbulences around topography irregularities such as scarps or rocks generate local noise and their proximity must be avoided.

Ocean generated noise: This is the most widespread noise (called microseisms or microseismic noise), and it is seen globally, although the interior of continents has less noise than coastal regions. Long period ocean microseisms are generated only in shallow waters in coastal regions (Borman, 2002), where the wave energy is converted directly into seismic energy either through vertical pressure variations, or smashing surf on the shores. They therefore have the same period as the water waves ($T \approx 10$ to 16 s). Shorter period microseisms can be explained by as being generated by the superposition of ocean waves of equal period traveling in opposite directions, thus generating standing gravity waves of half the period. These standing waves cause perturbations which propagate without attenuation to the ocean bottom. The higher frequency microseisms have larger amplitude than the lower frequency microseisms (Figures 3.3-3.5). During large storms, the amplitudes can reach 20000 nm at stations near the coast and make analog seismograms useless.

Other sources: Running water, surf and volcanic tremor (an almost harmonic noise associated to fluids motion, often lasting hours or days) or background activity are other local sources of seismic noise.

Man made noise and wind noise are usually the main source at high frequencies and since the lower limit is about 0.01 nm at 10 Hz, very small disturbances will quickly get the noise level above this value.

For more details on seismic noise generation, see Borman. (2002).