Q and spectral analysis in SEISAN

by

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The purpose of this note is to give the general background needed for understanding how Q is used and determined as well as some of the parameters used in spectral analysis. In addition references will be given to which SEISAN programs can be used for the different methods.

1.0 Attenuation

The amplitude attenuation caused by Q can be described as

$$A(f,t) = A_0 e^{-\frac{\pi f t}{Q(f)}} \qquad (1.1)$$

where A_0 is the initial amplitude, A(t) the amplitude after the wave has traveled time t, f is the frequency and Q(f) is the in general frequency dependent Q. Alternatively (1.1) is written

$$A(f,r) = A_0 e^{-\frac{\pi f r}{vQ(f)}} \qquad (1.1a)$$

where r is the hypocentral distance and v the average velocity along the path. For longer paths, the average velocity might change with hypocentral distance so is easier (and more correct) to use the travel time, which usually is a precisely known parameter for a given location and origin time.

If Q is constant along the path, (1.1) is all we need. If however Q varies along the path, the effect of the different parts of the path must be accounted for. For a two layer case (Figure 1), we get

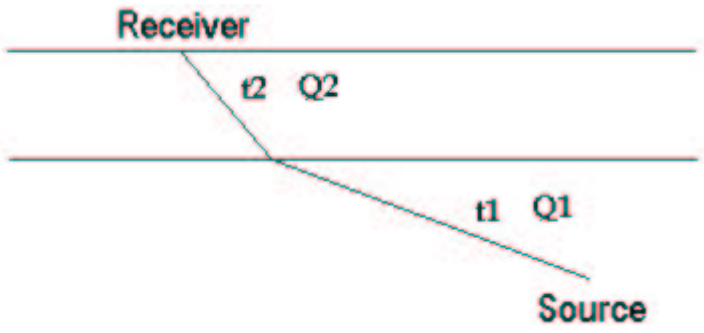

Figure 1 A ray passing two layers with different Q

$$A(f,t) = A_0 e^{-\frac{\pi f t_1}{Q_1(f)}} e^{-\frac{\pi f t_2}{Q_2(f)}} = A_0 e^{-\pi f \left(\frac{t_1}{Q_1(f)} + \frac{t_2}{Q_2(f)}\right)} \quad (1.2)$$

For a continuous changing Q we can write

$$A(f,t) = A_0 e^{-\pi f \int_{path} \frac{tdt}{Q(f)}} = A_0 e^{-\pi f t^*} \quad (1.3)$$

where t^* is defined as

$$t^* = \int_{path} \frac{tdt}{Q(f)} = \frac{T}{Q_{av}(f)} \quad (1.4)$$

and the integration is along the whole path, T is the total travel time along the path and Q_{av} is average Q along the path. For teleseismic body waves, t^* is often nearly constant for different travel times, due to increasing Q with depth, and independent of frequency (below 1 Hz) and therefore no frequency dependence is indicated. For ray paths used in local seismology, Q is most often considered constant along the ray path, although some increase of Q with depth has been observed. However, this turns out to be an over simplification since the near surface layers (1-3 km) generally have a much lower Q than the rest of the path and tends to filter out high frequency energy (f > 10-20 Hz). Thus for local crustal studies, we must at least separate the attenuation in two terms (near surface and the rest) as in (1.2) with a constant Q in each layer. It turns out that the near surface attenuation is nearly frequency independent, limited to the very near surface layers (similar travel time) and the effect can therefore be quantified with t^*:

$$A(f,t) = A_0 e^{-\pi f t^*} e^{-\frac{\pi f t_1}{Q_1(f)}} \quad (1.5)$$

In general t^* is used for teleseismic ray paths so in order to avoid confusion, we will use $\kappa = t^*$ instead for near surface attenuation. Since $t_1 >> t_2$, we will replace t_1 with t and Q_1 with Q and the general expression for the amplitude decay to use in local studies is then

$$A(f,t) = A_0 e^{-\pi f \kappa} e^{-\frac{\pi f t}{Q(f)}} \quad (1.6)$$

It should be noted, that the near surface attenuation is not the only phenomena, which can affect the amplitude at a given site. The so-called site amplification if often observed, whereby amplitudes at certain frequencies are enhanced (rarely also reduced) due to local soil structure and/or topographic effects. It is rarely observed at good rock sites while common on sediment sites, and is most pronounced on horizontal components. This effect is hard to distinguish from the effect of κ at individual frequencies and it is advised to use vertical component recordings at rock sites if possible for Q-determination.
The soil amplification can be determined with SEISAN using the SPEC program.

2.0 Determination of Q using one station

It will be assumed that κ is frequency independent and Q is frequency dependent on the form

$$Q(f) = Q_0 f^{\alpha} \tag{2.1}$$

where α usually is in the range 0.5 to 1.0. Taking the natural logarithm of (1.6), we get

$$\ln(A(f,t)) = \ln(A_0) - \pi f \kappa - \pi f \frac{t}{Q(f)} \tag{2.2}$$

We will assume that the signals are generated by earthquakes following the Brune source model (Brune, 1970) and therefore have a displacement source spectrum S of the form

$$S(f) = \frac{K}{1 + (\frac{f}{f_0})^2} \tag{2.3}$$

where K is a constant which is proportional to the seismic moment and f_0 is called the corner frequency. The spectrum is constant for $f < f_0$ and decays as f^2 for $f > f_0$. For $f = f_0$ the amplitude has decayed to 0.5 relative to the flat level or on a log-log scale to the –3db point (0.7).

Determine κ

For a constant t, we can consider (2.3) to represent the spectral content of a signal having travel the time t. If the signal is generated by an earthquake and we only use the part of the spectrum where $f_0 > f$, plotting ln(A(f,t)) vs. f, will make a straight line with slope of $-\pi(\kappa+t/Q)$, if Q is frequency independent. If t/Q is small, the slope will be directly proportional to κ. Thus to reliable determine κ in this way, without knowing Q, short hypocentral distances should be used. If Q(f) is known, the spectrum can first be corrected for Q(f), and κ determined directly. A good test of the result is therefore to determine κ using different distances. If Q(f) is correct, the value of κ obtained should be the same.
A typical value of κ is 0.05. With a travel time of 1 sec, that corresponds to a Q of 20.
Program SPEC can determined κ for individual events by fitting (2.2) to a log-lin spectrum and also for a series of events and determining average κ-values.

Q and κ from spectral modeling

The complete shape of the of the observed spectrum is (1.6) multiplied by (2.3) giving

$$A(f,t) = A_0 K \frac{1}{1+(\frac{f}{f_0})^2} e^{-\pi f \kappa} e^{-\frac{\pi f t}{Q(f)}} \tag{2.4}$$

Given an observed spectrum, (2.4) can be modeled to determine f_0, κ, Q_0 and α This is not always so easy, and is best done if at least one of the parameters is known. The modeling can be done with program MULPLT. Note that if α=1, the Q-term has no influence on the spectral shape since (2.3) becomes

$$A(f,t) = A_0 e^{-\pi f \kappa} e^{-\frac{\pi f t}{Q_0 f}} = A_0 e^{-\pi f \kappa} e^{-\frac{\pi t}{Q_0}} \qquad (2.5)$$

An α-value near 1.0 is quite common, particularly for coda Q values (see below).

If the corner frequency is high and if the effect of Q(f) is not dominating the spectrum, the near surface attenuation will dominate the spectral decay and the real corner frequency cannot be seen, only the apparent corner created by the near surface attenuation. If we define the corner frequency f_κ the frequency where the spectral level has reached 0.5 as a result of the effect of near surface attenuation, then f_κ can be calculated as

$$e^{-\pi \kappa f_\kappa} = 0.5 \quad \text{giving} \quad f_\kappa = \frac{0.223}{\kappa} \qquad (2.6)$$

If e.g. κ=0.025, f_κ=9 Hz, and it will not be possible to obtain the true source corner frequency for small earthquakes ($M_L < 3.0$) without correction for κ. It might not be possible to correct for κ, if the signal to noise ratio of the high frequency part of the spectrum is low due to the effect of κ. Thus in order to get a reliable source corner frequency for small earthquakes, κ is a very critical parameter.

A similar effect will be obtained with Q, however since Q normally is frequency dependent, the effect will not be as dramatic, more like a flattening of the spectrum. And, as observed above, if α is near 1, no change in spectral shape occurs. We can similarly define a f_Q as f_κ and get

$$e^{-\pi \frac{f_Q t}{Q_0 f_Q^{\alpha}}} = e^{-\pi \frac{f_Q^{1-\alpha}}{Q_0}} = 0.5 \text{ giving } f_Q = \left(\frac{0.223 Q_0}{t}\right)^{\frac{1}{1-\alpha}} \qquad (2.7)$$

For Norway, $Q=440f^{0.7}$ which gives f_Q=95 Hz if t=25 s. In this case, the Q-correction is not critical for determination of f_0.
For small (M= -1 to 1) very local earthquakes at Deception Island, $Q=59f^{0.4}$ and travel time is 2s giving f_Q=13 Hz, so obviously the q-correction is critical to get the correct corner frequency.

Coda Q

Coda waves are thought to decrease in amplitude only due to attenuation (1.6) and geometrical spreading and (1.6) can then be written

$$A(f,t) = t^{-\beta} A_0 e^{-\pi t \kappa} e^{-\frac{\pi f t}{Q(f)}} \qquad (2.8)$$

where β is 1 for body waves and 0.5 for surface waves. Usually it is assumed that coda waves are body waves. Taking the logarithm, (2.8) can be written

$$\ln(A(f,t) = \ln(A_0) - \beta \ln t - \pi f \kappa - \frac{tf\pi}{Q(f)} \quad \text{or} \tag{2.9}$$

$$\ln(A(f,t) + \beta \ln t = (\ln(A_0) - \pi f \kappa) - \frac{tf\pi}{Q(f)} \tag{2.10}$$

Plotting the envelope of ln(A(f,t) +βln(t) as a function of t for a given frequency (by band pass filtering the signal), gives a straight line with slope –πf/Q(f) and Q(f) can be determined. As it can be seen, the Q-determination is not affected by κ or soil amplification. Coda Q can be determined with program CODAQ.

Q of S and coda waves

From (2.8), the coda wave spectrum at time t_c can be written

$$A_c(f,t_c) = t_c^{-\beta} A_{c0} S(f) e^{-\pi f \kappa} e^{-\frac{\pi f t_c}{Q_c(f)}} \tag{2.11}$$

where A_{c0} is a factor depending on the efficiency of coda wave generation. It is assumed that the frequency content in the coda wave spectrum is the same as the S-wave source spectrum. The corresponding S-wave spectrum is

$$A_s(f,t_s) = t_s^{-\beta} S(f) e^{-\pi f \kappa} e^{-\frac{\pi f t_s}{Q_s(f)}} \tag{2.12}$$

where t_s is the S-wave travel time. We assume that the near surface attenuation and geometrical spreading is the same for coda waves and S-waves. The natural logarithm of the ratio of the two spectra is then

$$\ln(\frac{A_s(f,t_s)}{A_c(f,t_c)}) = -\beta \ln(\frac{t_s}{t_c}) - \ln(A_{c0}) - \pi f (\frac{t_s}{Q_s(f)} - \frac{t_c}{Q_c(f)}) \tag{2.13}$$

If Q is assumed frequency independent, and the spectral ratio is plotted as a function of f, we get a linear relationship with a slope of

$$\pi(\frac{t_s}{Q_s} - \frac{t_c}{Q_c}) \tag{2.14}$$

Alternatively, we can write (2.13) as

$$\ln(\frac{A_s(f,t_s)}{A_c(f,t_c)}) + \frac{\pi f t_c}{Q_c(f)} = -\beta \ln(\frac{t_s}{t_c}) - \ln(A_{c0}) - \pi f \frac{t_s}{Q_s(f)} \tag{2.15}$$

and the slope is

$$\pi \frac{t_s}{Q_s} \tag{2.16}$$

Since the travel times are known, either of the Q-values can be determined from the other. If Q_c is assumed frequency dependent, a constant Q_s can still be determined from the slope of the straight line using (2.15), similarly if Q_s is known and Q_c is unknown.

If Q(f) is to be determined, data from several stations and/or events are needed. Rearranging (2.15), we get

$$\ln(\frac{A_s(f,t_s)}{A_c(f,t_c)}) - \frac{t_c}{Q_c(f)} + \beta \ln(\frac{t_s}{t_c}) = -\ln(A_{c0}) - \pi f (\frac{t_s}{Q_s(f)}) \tag{2.17}$$

If, at one particular frequency, we have several observations at different t_s, it is seen from equation (2.17), that plotting the left hand side of (2.17) versus t_s, we get a linear relationship and $Q_s(f)$ can be determined from the slope.

All of the above methods will work without knowing the instrument calibration function, since spectral ratios form the same trace is used. There will be no dependence on focal mechanism if we assume that the spectral source content in S and coda waves are affected equally by the fault plane solution.

3.0 Two station methods for Q-determination

The principle is that if the waves are recorded at two different stations at different distances, the difference in amplitude, at a given frequency, is due to attenuation and geometrical spreading like in the coda Q case. We can write

$$A_1(f,t_1) = t_1^{-\beta} A_0 e^{-\pi f \kappa} e^{-\frac{\pi f t_1}{Q(f)}} \tag{3.1}$$

$$A_2(f,t_2) = t_2^{-\beta} A_0 e^{-\pi f \kappa} e^{-\frac{\pi f t_2}{Q(f)}} \tag{3.2}$$

for the amplitudes recorded at station 1 and 2 respectively. If we observe the amplitudes at specific frequencies and travel times, assume a geometrical spreading, assume that κ is constant, disregard the effect of focal mechanism, then the amplitude ratio at a given frequency at the two stations can be used to calculate Q(f).

$$\frac{A_2(f,t_2)}{A_2(f,t_2)} = (\frac{t_2}{t_2})^{-\beta} e^{-\frac{\pi f (t_2 - t_1)}{Q(f)}} \tag{3.3}$$

As it can be seen, Q(f) is the only unknown in (3.3). This method is critically dependent on the absolute amplitudes so, if instrument response is not the same at both stations, the amplitudes must be corrected for instrument response. Soil amplification can also seriously affect the results for individual values of Q(f). In a given region, κ is often constant and the effect of the source radiation can be eliminated by requiring that the station and the

source are on one line. Alternatively, we can use Lg waves which are not very sensitive to the focal mechanism (true ??). This method is used in program SPEC.

If Q is assumed independent of frequency, the ratio of A_2 and A_1 can be written

$$\frac{A_2(f,t_2)}{A_2(f,t_2)} = (\frac{t_2}{t_2})^{-\beta} e^{-\frac{\pi f (t_2 - t_1)}{Q}} \quad \text{and taking the logarithm} \qquad (3.4)$$

$$\ln(\frac{A_2(f,t_2)}{A_2(f,t_2)}) = \beta \ln(\frac{t_2}{t_2}) - \frac{\pi f (t_2 - t_1)}{Q} \qquad (3.5)$$

In this case, the spectral ratio is linearly related to the frequency so Q can be determined from the slope of the curve $\ln(A_2/A_1)$ versus f. This method is currently not implemented in SEISAN

If κ is not constant, the spectra will have to be corrected for κ before analysis or κ will have to be determined in a multiple station-event inversion, see below. Note that we still assume that all site dependent effects are included in κ.

Multiple station method for Q-determination

It as assumed, that we have k events recorded at l stations. Equation (2.9) can then be written as a series of equations

$$\ln(A_{kl}(f,t_{kl}) = \ln(A_{0k}) - \beta \ln t_{kl} - \pi f \kappa_l - \frac{t_{kl} f \pi}{Q(f)} \qquad (3.6)$$

For a given frequency, there will be one equation for each station-event pair. The unknowns are k source terms A_{0k}, l site terms κ_l and Q(f). This set of linear equations can be solved with standard methods and in addition to Q(f), also the site terms are determined. The geometrical spreading term might have to be replaced by a more detailed term, see section below. However, this does not change the set up and solution to the equations.

What about effect of source radiation, assumed averaged out ?
Program QLG in SEISAN is using this method.

4.0 Geometrical spreading in SEISAN

In the above description, it was assumed that geometrical spreading could be described on the form

$$A(t) = A_0 t^{-\beta} \text{ or equivalent } A(r) = A_0 r^{-\beta} \qquad (4.1)$$

where r is the hypocentral distance:

$$r = \sqrt{h^2 + x^2}$$

and h is the hypocentral depth and x the epicentral distance.

This formulation assumes a constant type of geometrical spreading independent of hypocentral distance. For S-waves, body waves are often assumed for the near field (β=1) and surface waves for larger distances (β=0.5) under the assumption that the S-waves are dominated by Lg waves and (4.1) is commonly written

$$A(r) = \frac{1}{r} \quad \text{for } r < r_0 \qquad (4.2)$$

$$A(r) = \sqrt{\frac{1}{rr_x}} \quad \text{for } r \geq r_0$$

where r_0 often is around 100 km. This works fine for surface focus events and S-Lg waves. However, in all the methods presented above, there is no limitation to use only S-Lg waves and shallow events, so (22) is not general enough. If e.g. an S-wave is recorded from a deep earthquake at more than r_0 epicentral distance, it would not be appropriate to use surface wave spreading. Thus in SEISAN, the geometrical spreading will also depend on the hypocentral depth and type of wave. Until depth h_1, and at larger distances, surface wave spreading is assumed, below depth h_2, body wave spreading is assumed and in between, an interpolation is made. In SEISAN we have thus defined a modified version of (22). The geometrical spreading is defined as

$$A_g(r) = \frac{1}{g_d} \qquad (4.3)$$

where g_d is called the geodistance defined as

P- waves:	$g_d = r$	for any x and h
S- waves:	$g_d = r$	for $x < x_0$ and any h
	$g_d = r$	for any x when $h \geq h_2$
	$g_d = \sqrt{xx_0}$	for $x \geq x_0$ and $h<h_1$
	$g_d = (1 - \frac{h-h_1}{h_2-h_1})\sqrt{xx_0} + \frac{h-h_1}{h_2-h_1} r$	for $x \geq x_0$ and $h_1 \leq h < h_2$

The parameters h_1 ,h_2 and x_0 can be set in SEISAN.DEF. The default values are h_1=50 km, h_2=100 km and x_0=100 km. There is not a specific scientific reason for (4.3), however it does provide a smooth transition between surface and body wave spreading as a function of depth and distance and it includes the earlier used relations (4.2).
Currently, only MULPLT is using geo distance in spectral analysis.

5.0 Seismic source model as used in Seisan

The Brune source model (Brune, 1970) is used which will give the following observed displacement spectrum at a distance r (m) and depth h (m), where the travel time is t

$$D(f,t) = \frac{M_0 * 0.6 * 2.0}{(1+(\frac{f}{f_0})^2)4\pi\rho v^3} A_g(r,h) e^{-\pi f \kappa} e^{-\frac{\pi f t}{Q(f)}} \quad (5.1)$$

where M_0 (Nm) is the seismic moment, the factor 0.6 accounts for average radiation pattern effect, the factor 2.0 is the effect of the free surface, ρ is the density (kg/m^3) and v is the velocity (m/s) (path ?????) at the source (P or S-velocity depending on spectrum). In SEISAN parameter files, the units are g/cm^3 and km/s. The calculated spectrum D_c, is usually just corrected for attenuation and the corner frequency f_0 and spectral flat level Ω_0 (ms) are the observed parameters:

$$D_c(f,t) = \frac{\Omega_0}{(1+(\frac{f}{f_0})^2)} = \frac{M_0 * 0.6 * 2.0}{4\pi\rho v^3 (1+(\frac{f}{f_0})^2)} A(r,h) \quad (5.2)$$

and the seismic moment can then be calculated as

$$M_0 = \frac{\Omega_0 4\pi\rho v^3}{0.6 * 2.0 * A(r,h)} \quad (5.3)$$

In case of simple 1/r body wave spreading (r in m), (5.3) would be

$$M_0 = \frac{\Omega_0 4\pi\rho v^3 r}{0.6 * 2.0} \quad (5.4)$$

In the literature, the effect of the average correction for radiation pattern varies between 0.55 to 0.85. According to Aki and Richards, the average is 0.52 and 0.63 for P and S-waves respectively. The effect for the free surface assumes a vertical incidence, which is an approximation. However, due to the low velocity layers near the surface, the incidence is not far from vertical. The effect is the same for P and S-waves.

Nothing has been mentioned about which component to use. The original Brune spectrum assumed SH waves so that should indicate that horizontal components should be used. On solid rock, experimental studies show that, for S or Lg-waves, there is little difference between the amplitude on the 3 components. However near surface amplifications is common (as determined with the SPEC program) which mainly affects the horizontal components. So it is safest to always use the vertical component which is any case is what should be used for the P-wave spectrum. Some studies use an average of all 3 components when making S-wave spectra.

The source radius a, is calculated as

$$a = 0.35 v_s / f_0 \qquad (5.5)$$

where the radius is m or km if the velocity is m/s or km/s

The factor used here (0.35) also can have various values. Brune (1970) uses a value of 0.37 for S-waves, Maderiaga (1976) uses 0.32 for P-waves and 0.21 for S-waves. In SEISAN, we have not assumed that P and S-waves can have different corner frequencies although this has been observed in some studies (e.g. Abercrombie, 1995) and predicted theoretically (Maderiaga, 1976).

The stress drop in bars (1 bar=10^6 dyne/cm^2) is calculated as (Eshelby,1957)

$$\Delta\sigma = \frac{7}{16} M_0 \frac{1}{a^3} * 10^{-14} \qquad (5.6)$$

Since the original formula assumes that moment is in dyne-cm and radius in cm the conversion factor 10^{-14} is needed for MKS units and is calculated as

((10^7 dyne cm /Nm)*1 /(10^5cm/km)^3)/ 10^6(dynes/cm^2)/bar = 10^{-14} since

1 Nm = 10^5dynes * 100 cm = 10^7 dyne-cm.

Self similarity

The generally accepted theory of self similarity predicts a constant stress drop for earthquakes of different size in the same tectonic environment. From (5.6) and (5.5) we then get

$$M_0 \approx a^3 \approx f_0^{-3} \qquad (5.7)$$

or in terms of moment magnitude M_w

$$M_w \approx \frac{\log(M_0)}{1.5} \approx -2\log(f_0) \text{ or} \qquad (5.8)$$

$$\log(f_0) \approx -0.5 M_w \qquad (5.9)$$

In terms of stress drop we can write

$$\Delta\sigma = \frac{7}{16} M_0 \frac{1}{a^3} 10^{-14} = \frac{7*10^{-14}}{16*(0.35 v_s)^3} M_0 f_0^3 \qquad (5.10)$$

or

$$\log(\Delta\sigma) \approx \log(M_0) + 3\log(f_0) \approx 1.5M_w + 3\log(f_0) \quad (5.11)$$

If the near surface attenuation is not accounted for, the corner frequency might be close to constant or not increasing fast enough when the magnitude goes down so plotting the logarithm of the stress drop versus M_w will give a straight line with slope 1.5. Alternatively relation (5.9) can be checked. Program MAG can be used for both of these plots.

Magnitude- corner frequency scaling relationship for the constant stress drop model

The moment magnitude is calculated as

$$M_w = \frac{\log(M_0)}{1.5} - 6.06 \quad (5.12)$$

Combining with

$$\Delta\sigma = \frac{7*10^{-14}}{16*(0.35v_s)^3} M_0 f_0^3 \quad \text{or} \quad (5.13)$$

$$M_0 = \frac{16\Delta\sigma * 0.35^3 v_s^3 10^{14}}{7 f_0^3} \quad (5.14)$$

gives

$$M_w = \frac{\log(16/7) + 3\log(0.35) + 3\log(v_s) - 3\log(f_0) + \log(\Delta\sigma) + 14}{1.5} - 6.06 \quad (5.15)$$

$$M_w = 2.90 + 2\log(v_s) - 2\log(f_0) + 0.67\log(\Delta\sigma) \quad \text{and} \quad (5.16)$$

$$\log(f_0) = 1.45 + 0.33\log(\Delta\sigma) + \log(v_s) - 0.5M_w \quad (5.17)$$

Assuming a stress drop of 10 bar and a S-velocity of 3.5 km/s gives

$$M_w = 4.67 - 2\log(f_0) \quad \text{or} \quad (5.18)$$

$$\log(f_0) = 2.00 - 0.5M_w \quad \text{stress drop 1 bar} \quad (5.19)$$
$$\log(f_0) = 2.23 - 0.5M_w \quad \text{stress drop 5 bar}$$
$$\log(f_0) = 2.33 - 0.5M_w \quad \text{stress drop 10 bar}$$
$$\log(f_0) = 2.66 - 0.5M_w \quad \text{stress drop 100 bar}$$

This relation can be compared to a similar relation derived by Eaton (1977) (taken from Lee and Stewart, 1981) for M_L assuming a stress drop of 5 bars.

$$\log(f_0) = 2.1 - 0.5M_L \tag{5.20}$$

As we see, it is almost identical to the above M_w relation for stress drop 5 bars. These relations should at least give a hint whether an observe corner frequency is 'reasonable' or when working with small earthquakes, whether near surface attenuation has severely limited the high frequency resolution.

Expected scaling between M_L and M_w for small earthquakes

The amplitude used to determine M_L is determined on the flat part of the spectrum and is therefore expected to be linearly proportional to Ω_0 and therefore M_0. So M_L must be linearly proportional to $\log(M_0)$ and we get

$$M_w = \frac{\log(M_0)}{1.5} - 6.06 \approx \frac{M_L}{1.5} \tag{5.21}$$

or

$$M_L \approx 1.5M_w \tag{5.22}$$

References

Abercrombie, R. E. (1995) Earthquake source scaling relationships from –1 to 5 M_L using seismograms recorded at 2.5 km depth, J. Geophys. Res, 100, 24015-24036.

Aki, K. and P. G. Richards (1980). Quantitative seismology – theory and methods. W. H. Freeman and Company, San Francisco.

Brune, J (1970). Tectonic stress and seismic shear waves from earthquakes, J. Geophys. Res., 75, 4997-5009.

Eaton, J. P. (1977). Frequency response of the USGS short period telemetered seismic system and its suitability for network studies of local earthquakes. USGS open file report 77,844.1-45.

Eshelby, J. D. (1957). The determination of the elastic field of an ellipsoidal inclusion and related problems, Proc. R. Soc. London A, 241, 376-396.

Lee, W. H. K. and S. W. Stewart (1981). Principles and applications of microearthquake networks, Academic press.

Maderiaga, R (1976). Dynamics of an expanding circular fault. Bull. Seism. Soc. Am. 66,639-666.